Where ignorance is bliss, 'tis folly to be wise - the value of information in contests*

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– do not quote –

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Abstract

We analyze a two-player Tullock contest with asymmetric valuations and one-sided asymmetric information. First, we give a general characterization of the possible equilibria arising in the Bayesian game. Then we allow for information acquisition, which may be observable or not. We show that the uninformed player might be willing to pay in order to stay uninformed when information acquisition is perfectly observable. In those situations she can use her ignorance as a strategic instrument; ignorance has a value. If information acquisition is not perfectly observable the unique equilibrium in pure strategies is characterized by spying and playing full information Nash equilibrium efforts.

Keywords: Contest, Asymmetric Information, Commitment

JEL-Classification: D72, D82, L12

* "Where ignorance is bliss, 'tis folly to be wise" is taken from from Thomas Gray's poem "Ode on a Distant Prospect of Eaton College". We are indebted to Stefan Bühler, Magnus Hoffmann, and Martin Kolmar for very helpful suggestions and comments. Of course, all errors remain our own.

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1 Introduction

Assume an advertising intense market, as for example the market for cars or newspapers. It is well known that competition in such markets can be described adequately by a contest.[1] Or take political competition in a democracy. (Potential) candidates woo for voters' or a parties' support in order to get elected. This is another example of a contest. Both types of contest are well known as long as all players' attributes are common knowledge. However, as soon as a new player enters the game, be it a yet unknown firm or a young and yet unknown politician,[2] we should expect informational asymmetries in such a game. In this paper we address this issue in detail and show how the information structure influences outcomes. Specifically, we look at a Tullock contest where 2 players compete for some prize. Players are heterogeneous with respect to their valuation. The valuation of one player, the incumbent, is common knowledge. The entrants valuation of the prize is her private knowledge and only the distribution is common knowledge. In this framework we first solve for all equilibria and identify conditions for corner solutions. Second, and more surprisingly, we identify conditions under which the incumbent player is actually better off without knowing the true type of her opponent. This finding seems to be new in the literature.

During the past 30 years there have been two prevalent contest models, perfectly discriminating contests (or, in the parlance of auction theory: all-pay auctions) and imperfectly discriminating Tullock contests. Both types have been employed widely to model various situations, such as rent seeking, political campaigns, or R&D races.[3] For an excellent and comprehensive recent overview over the literature on single-prize contests consult Konrad (2009). Sisak (2009) very recently surveys the literature on multiple prizes.

Here we focus on Tullock contests. We show how a player can use her ignorance about the opponent's type as a commitment. The role of commitment in games is well known at least since von Stackelberg (1934). Usually, it is a strategic advantage if a player can

[1] See, for example, Friedman (1958), Schmalensee (1976), or Rosen (1988). Another paper discussing competition through a contest is Konrad (2000). He argues that in many markets, for example in the European insurance market before 1994, when deregulation took place, prizes were strictly maintained by the authorities. Companies competed for contracts by letting their agents spend time and money to approach potential customers and built up a relationship. This form of competition is another example of a contest. In this framework he analyzes the optimal location decision of two competing firms not knowing where exactly the customers are located on the unit interval.

[2] A recent example in politics is Barack Obama, who, though still being quite unknown to the public in the U.S., entered the democrats' competition for being a presidential candidate in the 2008 election.

[3] See, for example, Tullock (1980), Hillman and Riley (1989), and Ellingsen (1991) on rent seeking, Konrad (2004) and Klumpp and Polborn (2006) on political campaigns, Baye and Hoppe (2003) on R&D races or Kolmar and Sisak (2008) on shaping optimal incentives to provide a public good.

move *first* because this allows her to commit to a particular action. The intuition for this advantage comes from the fact, that the player who moves first can *commit* to choose an action/strategy which is not in her own best-response correspondence, however, taking into account the best response of all other players. For example a Stackelberg leader in a Cournot oligopoly game chooses her output, taking into account the response of the followers, and so can gain a bigger profit and market share. In the the literature on contests Dixit (1987) found, that this finding carries over to a two player Tullock contest. A player who can move first has an advantage, because she can commit to a particular level of effort. Depending on the relative abilities of the contestants, the leader's effort level possibly differs from the effort level in a full-information Nash equilibrium. In particular, if the leader is the favorite (underdog) of the game, she will spend more (less) than in Nash equilibrium.[4] The intuition is the following: in the Nash equilibrium the favorite's efforts are strategic substitutes to the underdogs efforts, whereas the underdogs efforts are strategic complements to the favorite's efforts. Therefore, by committing to a higher (lower) effort, the favorite (underdog) can induce lower equilibrium efforts of their opponents. Baik and Shogren (1992) generalized Dixit's analysis and endogenized the order of moves. They find, that only the underdog will in equilibrium act as the Stackelberg leader. In their framework the underdog can commit to a very low level of effort, and since the underdogs efforts are a strategic complement to the favorite's effort the favorite will also cut down her effort level, compared to the Nash outcome. Hence both players can save on efforts and the overall intensity of the contest is reduced, making both players better off. Yildirim (2005) adds multiple stages to Baik's and Shogren's framework. Interestingly this generalization leads to the opposite findings. Now the favorite is the only player that will act as a leader. The intuition for this is, that now the underdog cannot commit to lower levels of effort anymore. She always has the opportunity *and* the incentive to increase her efforts to the Nash equilibrium levels in the last stage. On the other hand, the favorite, who as Stackelberg leader spends more than in the Nash equilibrium, cannot cut down her efforts in the last stage. Fu (2006) endogenizes the order of moves in Tullock contests with one-sided asymmetric information. He finds, that sequential equilibria always exist and the uninformed player has an incentive to move first, in order to soften the competition.

In our paper we focus on another dimension of strategic commitment in Tullock contests,

[4]The terms 'favorite' and 'underdog' are due to Dixit (1987). A given player is called the favorite (underdog) if her equilibrium winning probability is larger (smaller) than 50 percent.

which is due to the presence of asymmetric information.[5] The papers most closely related to ours are Hurley and Shogren (1998b), Baik and Shogren (1995), Morath and Münster (2008), Slantchev (2008), and Morgan and Várdy (2007). Baik and Shogren (1995) consider an asymmetric information contest, where players' utility functions exhibit decreasing aversion to uncertain ability. They come to the finding, that under certain conditions information asymmetries increase rent dissipation as compared to a full information scenario. However, if spying is possible rent dissipation declines again. Hurley and Shogren (1998b) discuss equilibria in two-player Tullock contests with one-sided asymmetric information. Slantchev (2008) looks at a two-player contest with one-sided asymmetric information, where the type of the player with private information is drawn from a two-point distribution. In this context he discusses equilibria where all types or only one type of the unknown player are active. Morath and Münster (2008) analyze information acquisition in the context of All-pay auctions, where player do not know there own type and the type of their opponent. Each player can engage in acquiring the information about her own type. Depending on the cost of information acquisition only one player might invest in information. Therefore, one-sided asymmetric information can emerge *endogenously* in contests. Morgan and Várdy (2007) analyze a Stackelberg Tullock contest, where the follower has to pay in order to observe the leaders move. They find, that all Stackelberg equilibria break down and all players choose their respective Nash equilibrium (pure) strategies in equilibrium.

We analyze a simultaneous move Tullock contest with two players and one-sided asymmetric information, as discussed in Hurley and Shogren (1998b). In our model player 1's valuation of the prize is common knowledge, whereas player 2's valuation is drawn from a distribution function with support $[a, b] \subseteq \mathbb{R}^+$. First we generalize Hurley and Shogren (1998b)'s analysis and provide closed form solutions for both interior and corner solution equilibria. Then we extend our analysis and allow for costless information acquisition. The informed player receives a signal about whether the uninformed player acquired information. However, this signal is possibly noisy or completely uninformative. We find that when information acquisition is perfectly observable the uninformed player might have an incentive to voluntarily abstain from doing so. This is because her ignorance has in some situations a strategic value.[6] However, when information acquisition is not perfectly observable both players always choose

[5]The literature analyzing the consequences of informational asymmetries in contests is rapidly growing. Interesting papers, that are not directly related to our analysis, are Hurley and Shogren (1998a), Wärneryd (2003), Malueg and Yates (2004), Schoonbeek and Winkel (2006), and Fu (2006).

[6]There are a lot of other papers identifying a value of ignorance in various situations, for example Barros (1997), Kessler (1998), or Schmitz (2007). See also section 5.

the same efforts as in the Nash equilibrium, and the uninformed player always spies. This is in line with the literature focusing on imperfectly observable commitment in Stackelberg games, initiated by Bagwell (1995)'s noisy leader game, which also motivated Morgan and Várdy (2007)'s analysis. However, while this literature focuses on simple Stackelberg games, where the follower observes the leaders strategy choice only subject to some noise, we model a multistage game, where the contest itself is a simultaneous move game. In contrast to Morath and Münster (2008), we do not need positive costs of spying, but full observability of spying. In contrast to Baik and Shogren (1995) we focus on conditions under which players want to acquire information in equilibrium or not.

The paper is organized as follows. In the next section we shortly review standard results from full information Tullock contests as a benchmark. In section 3 we proceed to analyze the contest when there is one-sided asymmetric information with respect to the valuation of player 2. In section 4 we develop a set up to analyze the game when information acquisition is possible. In section 5 we solve the model for the case of perfectly observable information acquisition, in section 6 when information acquisition is not perfectly observable. In section 7 we provide a short application of the model in the context of a market entry game. In section 8 we briefly discuss recent papers analyzing signalling in Tullock contests, what might serve as an alternative means for information transmission. Section 9 concludes.

2 Full Information: The benchmark case

Assume a standard two player Tullock contest where player $i = 1, 2$ spends effort x_i in order to win a prize to which he assigns a value v_i. Costs are linear and marginal cost are unity. We abstain from modeling the players heterogeneous with respect to their costs, to keep the analysis as simple as possible and to isolate the commitment effect in the next section. A general discussion with all important results in this benchmark case can be found in Konrad (2009).

Player i maximizes her expected utility in the full information (FI) game

$$\max_{x_i \in \mathbb{R}^+} EU_i^{FI} = Pr_i[\text{win}]v_i - x_i = \frac{x_i}{x_i + x_j} v_i - x_i \quad (1)$$

where $Pr_i[\text{win}] = 1 - Pr_j[\text{win}]$, $i = 1, 2$ and $i \neq j$. The individual best responses are single-valued and given by

$$x_i^*(x_j) = \max\{0, \sqrt{v_i x_j} - x_j\}. \quad (2)$$

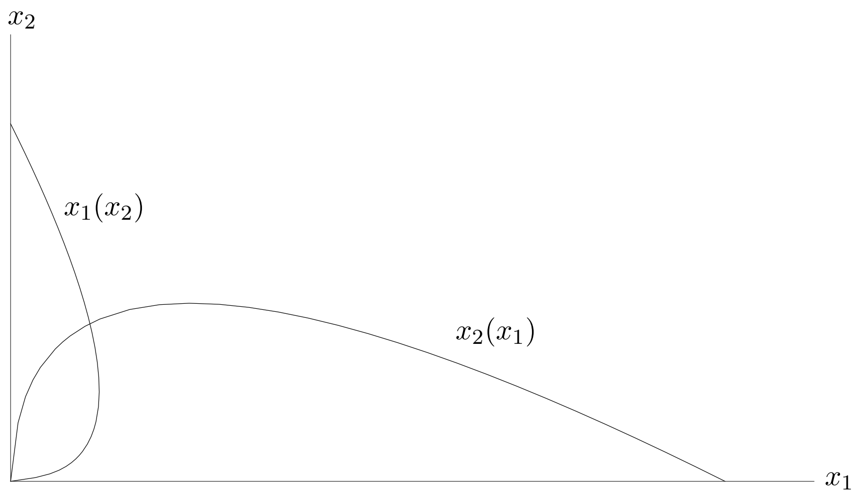

Figure 1: Here we see the reaction functions of player 1 and player 2. Player 2 is the favorite, implying that at the point of intersection her reaction function's slope is positive (strategic complement), whereas the slope of player 1's reaction function is negative (strategic substitute).

Given the best response it is easily shown that the Nash equilibrium (NE) efforts are given by

$$x_i^{NE} = \frac{v_i^2 v_j}{(v_1 + v_2)^2} > 0. \tag{3}$$

Equilibrium efforts are monotonically increasing in the own valuation of the prize and are monotonically decreasing (increasing) in the valuation of the other player whenever the other player is the favorite (underdog). This is due to the fact, that the favorite's (underdog's) effort is a strategic substitute (complement) to the underdog's (favorite's) effort - a property that will become important later on (see also figure 1). In our setting the favorite is the player with the higher valuation, because she will spend more effort and hence has a higher probability of winning. Given the equilibrium strategies it is straightforward to compute the equilibrium expected utilities of the players:

$$EU_i^{NE} = \frac{v_i^3}{(v_1 + v_2)^2} > 0.. \tag{4}$$

This is strictly increasing (decreasing) in the own valuation (valuation of the opponent) and will be our benchmark case. In the following we turn to the analysis of a contests with asymmetric information.

3 Incomplete Information: Bayesian Equilibrium

We model the one-sided asymmetric information following Hurley and Shogren (1998b), except, that in our framework players are not heterogeneous with respect to the effectiveness of efforts. Player 1's valuation is common knowledge, whereas player 2 has private information about her valuation v_2. This valuation is drawn from the interval $[a, b] \subseteq \mathbb{R}^+$ according to the distribution function $G(v_2)$, and this is again common knowledge. Therefore the following is also a generalization of the "contest endgame" in Slantchev (2008), who is dealing with a two-point distribution.[7]

We now proceed to analyze the game in more detail. Player 1's optimization problem in the incomplete information game (II) is now given by

$$\max_{x_1 \in \mathbb{R}_0^+} EU_1^{II} = Pr_1[\text{win}]v_1 - x_1 = v_1 \int_a^b \left[\frac{x_1}{x_1 + x_2(v_2)} \right] dG(v_2) - x_1. \tag{5}$$

Her first order conditions are easily found and given by

$$\frac{\partial EU_1^{II}}{\partial x_1} = \int_a^b \left[\frac{x_2(v_2)}{(x_1 + x_2(v_2))^2} \right] v_1 dG(v_2) - 1 \geq 0. \tag{6}$$

For player 2, who holds full information, the problem is identical to before and therefore her best response is still given by (2). We can use this information to determine the efforts in the Bayesian Nash equilibrium, denoted by the superscript BE.

Proposition 1. *Given the game has an interior solution, in the unique Bayesian Nash equilibrium player 1's effort is*

$$x_1^{BE} = \left(\frac{v_1 \int_a^b z^{-0.5}\, dG(z)}{v_1 \int_a^b z^{-1}\, dG(z) + 1} \right)^2 > 0 \tag{7}$$

and player 2's equilibrium effort is given by

$$x_2^{BE} = \sqrt{v_2 x_1^{BE}} - x_1^{BE}. \tag{8}$$

Proof. See appendix. □

Sometimes it is convenient to use expectation operators instead of integrals, i.e. $x_1^{BE} = \frac{v_1^2 E[1/\sqrt{v_2}]^2}{(v_1 E[1/v_2]+1)^2}$. If we use the expectation operator the expression holds also when we are dealing

[7] In section 8 we discuss the connections between Slantchev (2008) and our paper.

with discrete or mixed distributions of types.[8]

For an interior solution it has to hold, that all types of player 2 invest positive effort in equilibrium. Thus, $\sqrt{v_2 x_1} - x_1 \geq 0$, or, equivalently, $v_2 \geq x_1$, has to hold. This is trivially fulfilled when player 1 is the clear underdog, i.e. $v_1 \leq a$. However, since player 1 will never spend her total valuation of the prize, the contest is noisy, an interior solution can also be found when $a < v_1$. From (2) we know, that the type of player 2 with $v_2 = a$ would be the first to run into a corner solution. Therefore, if this type will be active in the contest the game has an interior solution.

Corollary 1. *The game has an interior solution if and only if the following condition is fulfilled:*

$$v_1 \leq \tilde{v}_1 = \frac{\sqrt{a}}{\int_a^b \left[\frac{\sqrt{z}-\sqrt{a}}{z}\right] dG(z)}.$$

Proof. This follows immediately from proposition 1 and (2), given that $v_2 = a$. □

$\tilde{v}_1$ is strictly positive and, as we should expect, increasing in a. The higher a, the less likely it is that $x_1^{BE} > v_2$. If this condition is fulfilled (and only then) all types of player 2 will spend positive effort and thus we label this an interior solution. There are also scenarios in which only some of the player 2 types will be active in equilibrium. This will be the case when $v_1 > \tilde{v}_1$. Then, some of the player 2 types will not participate in the contest actively and hence we label this a corner solution. The following proposition characterizes the corner solution equilibria of this game, characterized by the superscript CS.

Proposition 2. *In the unique corner solution Bayesian Nash equilibrium player 1's effort is given by*

$$x_1^{CS} = \left(\frac{v_1 \int_{\bar{v}_2}^b z^{-0.5} dG(z)}{v_1 \int_{\bar{v}_2}^b z^{-1} dG(z) + 1} \right)^2 \tag{9}$$

where $\bar{v}_2 \in (a, b]$ is the marginal type of player 2 implicitly defined by

$$\bar{v}_2 = x_1^{CS}.$$

[8] Efforts here depend on *negative moments* of the distribution function $G(\cdot)$. Generally, a negative moment of a distribution is given by $E[x^p]$, where $p < 0$. Those moments do not necessarily exist when the support of the density includes zero. We do not encounter this problem since $0 < a < b$. Statistical articles dealing with the existence of such moments are for example Chao and Strawderman (1972), Piegorsch and Casella (1985), or Casella and Khuri (2002).

Player 2's equilibrium effort is given by

$$x_2^{CS} = \begin{cases} \sqrt{v_2 x_1^{CS}} - x_1^{CS} & \textit{if} \quad v_2 > \bar{v}_2 \\ 0 & \textit{else.} \end{cases}$$

Proof. See appendix. □

Player 1 will always spend strictly positive effort, however, less than in the equilibrium with an interior solution, $x_1^{CS} < x_1^{BE}$. The intuition for this is straightforward: if player 2 stays passive player 1 wins the whole prize for sure by spending some $\epsilon > 0$. All types of player 2 with $v_2 \leq \bar{v}_2$ will stay passive in the contest and spend zero effort. All types $v_2 > \bar{v}_2$ will invest strictly positive efforts. Corner solutions in Tullock contests are frequently encountered. For example, Grossman and Kim (1995) analyze equilibria of sequential Tullock contests with asymmetric players. In their setting, the leader is the defender of some resource and can invest in defense / fortification to secure her property. Depending on the relative strength of players the leader can fully deter any aggressive behavior. Technically, this is a corner solution, in which only one player invests in the contest. The effect is a similar to ours, however, the cause is a different one, since we analyze a simultaneous game. While in Grossman and Kim (1995) the order of moves might allow the leader to deter any action of the follower, in our setting the lack of information is the driving force, which allows player 1 to deter some types of player 2.

For the remainder of the paper we will assume that we are in an interior solution. The results for the corner solution case are qualitatively similar, except for the different distribution of active types and the utilities in equilibrium.

We are now able to calculate the equilibrium utilities for both players.

Corollary 2. *In the Bayesian equilibrium with interior solution player 1's expected utility is*

$$EU_1^{BE} = \frac{v_1^3 \left(\int_a^b z^{-1}\, dG(z)\right) \left(\int_a^b z^{-0.5} dG(z)\right)^2}{\left(v_1 \int_a^b z^{-1}\, dG(z) + 1\right)^2} \tag{10}$$

and player 2's expected utility is

$$EU_2^{BE} = v_2 + x_1^{BE} - 2\sqrt{v_2\, x_1^{BE}} \geq 0.$$

Proof. This follows immediately from (5), (1), and proposition 1. □

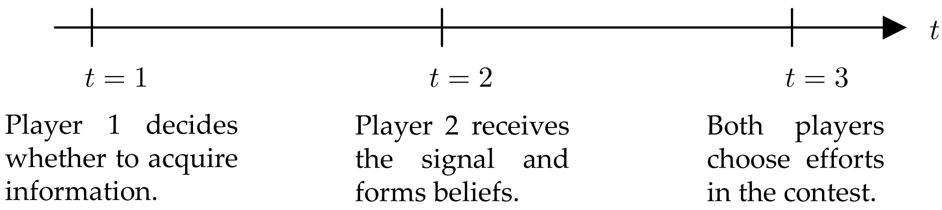

Figure 2: Sequence of moves in the information acquisition game.

4 Acquiring Information

As pointed out in Hurley and Shogren (1998b), one key difference between the complete information contest in section 2 and the asymmetric information contest just analyzed, is that player 1's equilibrium effort is not on her *ex-post* reaction function. Therefore, while ex-ante her effort choice is optimal, this is (most likely) not the case from an ex-post point of view. But does this imply, that player 1 is necessarily better off under full information? As we will show in the following the answer to this question is no. The reason is that she can commit to a level of efforts which puts her in a position similar to a Stackelberg leader. In this section we will address this issue in more detail.

In the first stage player 1 decides on whether to spy or not.[9] Then, in the second stage, player 2 receives a signal, which tells her which action player 1 has chosen before. This signal could be noisy or wrong and is not observable for player 1. Conditional on the signal player 2 forms beliefs. In the third and last stage both players simultaneously choose their efforts in the contest. The sequence of moves can also be seen in figure 2. Let the action of player 1 in stage 1 be denoted by $\alpha \in \{s, n\}$, and the signal player 2 observes be given by $\sigma \in \{s, n\}$, where s indicates that player 1 spied and n indicates that she did not. We allow for a noisy signal. Particularly, the probability to receive the correct signal is

$$Pr[\sigma = i | \alpha = i] \quad = \quad 1 - \epsilon, \quad \epsilon \in [0, 1/2], \; i = s, n. \tag{11}$$

The value of ϵ is common knowledge. We model the signal similar as in the noisy leader game in Bagwell (1995). For $\epsilon = 0$ player 2 can infer without any doubt what player 1 chose in stage 1, and thus we call the signal *informative*. However, if $\epsilon = 1/2$ the signal is completely worthless and we call it *uninformative*. For intermediate values of ϵ we say the signal is *noisy*.

A strategy for player 1 is a 2-tuple $A = \{\alpha, x_1\}$ specifying the action α in stage 1 as well

[9]We will use the term 'spy' when we talk about information acquisition, and we use this term throughout the paper.

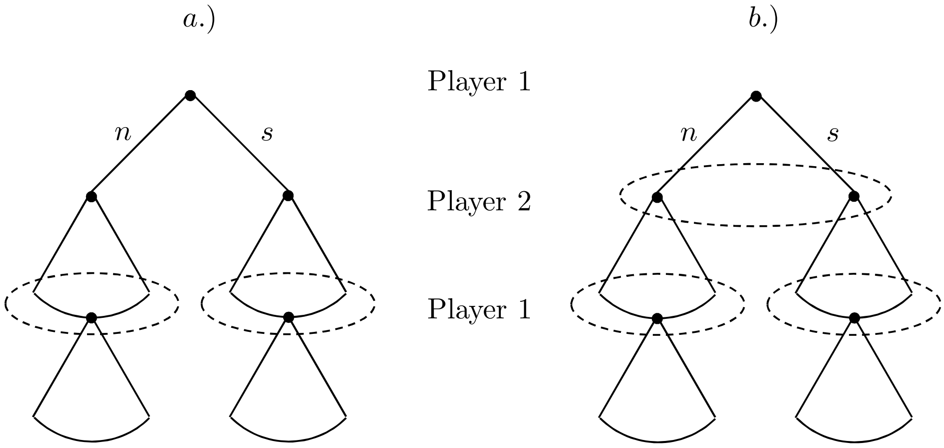

Figure 3: Panel a.) shows the game when $\epsilon = 0$, panel b.) when $\epsilon \in (0, 1/2]$.

as the effort level x_1 in stage 3. Player 2 forms beliefs

$$\beta(\sigma) = (\beta(s), \beta(n))$$

about player 1's stage 1 action after receiving the signal $\sigma \in \{s, n\}$ and her strategy specifies her effort in stage 3, $x_2(\sigma) = (x_2(s), x_2(n))$. Together, her belief and strategy form another 2-tuple $B = \{\beta(\sigma), x_2(\sigma)\}$. The equilibrium concept we employ is perfect Bayesian equilibrium and such an equilibrium is given when the players' strategies are sequentially rational given the beliefs and beliefs are consistent with the strategies.

5 Informative signal: perfectly observable information acquisition

We begin with the analysis of the game when the signal is informative ($\epsilon = 0$). The structure of this game can be seen in panel a.) of figure 3. Solving the game backwards player 1 knows the respective payoff in the two possible subgames following after his first stage decision, $\alpha \in \{s, n\}$. Acquiring information will induce a subgame of full information, as in the benchmark in section 2. Staying ignorant on the other hand leads to the subgame analyzed in section 3. She will compare the respective subgame payoffs and choose the action in stage 1 accordingly.

If she chooses to acquire information, player 1 gets as payoff the expectation of the full-information Nash equilibrium payoff in (4), which is given by

$$EU_1^{spy} = \int_a^b \left[\frac{v_1^3}{(v_1+z)^2} \right] dG(z). \tag{12}$$

If she abstains from acquiring information the game is one of one-sided asymmetric information and her equilibrium payoff is given by (10). We define the difference of those two payoffs as

$$D = \int_a^b \frac{v_1^3}{(v_1+z)^2} dG(z) - \frac{v_1^3 \left(\int_a^b z^{-1}\, dG(z) \right) \left(\int_a^b z^{-0.5} dG(z) \right)^2}{\left(v_1 \int_a^b z^{-1}\, dG(z) + 1 \right)^2}. \tag{13}$$

For simplicity we assume that player 1 chooses to acquire information when she is indifferent, so that $\alpha = s$ whenever $D \geq 0$.

Let us analyze D in more detail. The first term, i.e. the expected utility from spying, increases when probability mass is shifted from values close to b to values close to a. This is not necessarily true for the second term, i.e. the utility in the Bayesian game. Here are two partly opposing effects at work. On the one hand expected utility increases when probability mass is shifted from b to a, as before, because the expected opponent gets weaker. This is captured in $E[1/\sqrt{v_2}]$. However, there is another important effect. When the bulk of probability is centered around b, $E[1/v_2]$ is very small. Now, when we shift probability mass towards a this expectation increases. At the same time D decreases. However, this is true only as long as $E[1/v_2] > 1/v_1 \Leftrightarrow E[v_1/v_2] > 1$. When this condition is not fulfilled D increases in $E[1/v_2]$. Therefore, D is non-monotonic in $E[1/v_2]$. The following facts summarize the effects.

Fact 1. *D is strictly increasing in the second negative moment of $G(v_2)$, $E[(v_1+v_2)^{-2}]$. Moreover, $E[(v_1+v_2)^{-2}] \in \left(\frac{1}{(v_1+b)^2}, \frac{1}{(v_1+a)^2} \right)$.*

Fact 2. *D is strictly decreasing in $E[v_2^{-0.5}]$, which is a negative fractional moment. Moreover, $E[v_2^{-0.5}] \in \left(\frac{1}{\sqrt{b}}, \frac{1}{\sqrt{a}} \right)$.*

Fact 3. *D is U-shaped in the first negative moment of $G(v_2)$, $E[v_2^{-1}]$, and has a minimum at $E[v_2^{-1}] = 1/v_1$. This is equivalent to $E[v_1/v_2] = 1$. Moreover, $E[v_2^{-1}] \in \left(\frac{1}{b}, \frac{1}{a} \right)$ and thus, if $E[v_2^{-1}] = 1/v_1$ this also implies $b > v_1 > a$.*

Proof. Those facts follow immediately from inspection of (13). □

Those facts tell us conditions in which it is more or less likely that spying is optimal. Effects which decrease D make it more likely that not to spy is optimal. Since all effects depend on moments of the distribution of types we can characterize distributions under which staying passive is likely to be optimal, given v_1.

Note that fact 3 does not tell us D reaches a global minimum when $\int_a^b z^{-1}dG(z) = 1/v_1$ (see facts 1 and 2). However, fixing all other moments of the distribution and letting only $\int_a^b z^{-1}dG(z)$ vary, D has a local minimum when $\int_a^b z^{-1}dG(z) = 1/v_1$. This tells us, that it is ceteris paribus more attractive not to spy, when the expected ratio of valuations is close to unity, i.e., when there is no clear favorite. Note also, that $E[1/v_2] = 1/v_1$ implies $v_1 < E[v_2]$. This follows directly from Jensen's inequality, and tells us, that is is more likely to be optimal not to spy when the expected type of player 2 values the prize more than player 1, but there are still some types of player 2 with a valuation lower than v_1. Unfortunately we cannot prove existence of $D < 0$ generally. However, we can prove existence for the case of a continuous uniform distribution.

Fact 4. *If $G(v_2)$ is a uniform distribution on the interval $[a, b] \subset \mathbb{R}^+$ there always exist some v_1 for which $D < 0$.*

Proof. See appendix. □

Furthermore, we can show that if player 1's valuation is either very high or very low information acquisition is typically optimal.

Fact 5. *Whenever either $v_1 \to 0$ or $v_1 \to \infty$, $D > 0$.*

Proof. See appendix. □

The following proposition characterizes the equilibrium discussed above.

Proposition 3. *Let $\epsilon = 0$. If $D \geq 0$ the equilibrium of this game is given by*

$$\begin{aligned} A_{D\geq 0}^{\epsilon=0} &= \{s, x_1^{\epsilon=0} = x_1^{NE}\} \\ B_{D\geq 0}^{\epsilon=0} &= \{(\beta(\sigma) = (s,n), x_2^{\epsilon=0}(\sigma) = (x_2^{NE}, x_2^{BE})\}. \end{aligned}$$

If, however, $D < 0$, the equilibrium is given by

$$\begin{aligned} A_{D<0}^{\epsilon=0} &= \{n, x_1^{\epsilon=0} = x_1^{BE}\} \\ B_{D<0}^{\epsilon=0} &= \{(\beta(\sigma) = (s,n), x_2^{\epsilon=0}(\sigma) = (x_2^{NE}, x_2^{BE})\}. \end{aligned}$$

This equilibrium is the unique subgame-perfect Bayesian Nash equilibrium.

Proof. See appendix. □

What is the intuition for player 1 being better off without spying in some cases? As we have seen before when we discussed corner solutions, the deterrence effect is quite important. Let us look at this in more detail. First, as pointed out by for example Dixit (1987), in logit or Tullock contests the reaction functions of the different players are not-monotonic and hump shaped. Also, a given player's effort is maximal when her opponent is of the same strength, i.e., the reaction functions of two identical players have a unique intersection at their respective maximum.

Assume there are only two types of player 2 with respective valuations v_2^h and v_2^l with $v_2^l < v_1 < v_2^h$ and that player 1 is the expected favorite (her probability of success in the Bayesian equilibrium exceeds 50% in expectation). Then player 1's equilibrium effort in the Bayesian game is in between her Nash efforts and her effort is maximal in the Nash equilibrium against the high valuation type. So on the one hand she will be able to overcommit effort relative to the Nash equilibrium against the l-type. On the other hand, player 1 can undercommit relative to the Nash equilibrium against the high valuation type. It is well known since Dixit (1987), that in a Nash equilibrium the favorite's efforts in the contest are strategic substitutes to the underdog's efforts, and that the underdog's efforts are strategic complements to the favorite's efforts. Put differently, if the Nash equilibrium efforts are the reference point, higher efforts of the favorite *decrease* the underdog's efforts, while higher efforts of the underdog *increase* the favorite's efforts. Therefore, it is favorable for her not to spy because so she can induce lower efforts from both types of player 2.[10] Summarizing, in this scenario player 1 can benefit in both situations from not knowing the true type. This intuition carries over to some other and more general cases as well. For example, as we have seen in fact 4, in the case of a continuous uniform distribution on $[a, b] \subset \mathbb{R}^+$ there are always values of v_1 such that staying ignorant is worthwhile.

Although we think that our results are new in the context of the theory of contests, there are other fields of microeconomic theory in which similar results were found. For example, Barros (1997) analyzes an oligopolistic industry in which firm owners might have an incentive to stay uninformed about the operations of their agents. The reason is, that because of the information asymmetry the firm commits not to extract the agent's total surplus, what in turn gives the agent incentives to invest effort. But this in turn is also beneficial to the firm.

[10]The terms *strategic substitute* and *strategic complement* are due to Bulow, Geanakoplos, and Klemperer (1985). Generally, the decisions of two or more players in a game are called strategic complements if they mutually reinforce each other. They are called strategic substitutes if they mutually offset one another.

In the agency literature there are also papers focusing on the value of being ignorant. For example, Kessler (1998) analyzes an adverse selection framework in which an agent might have the incentive to acquire information about an economically relevant parameter, like the costs of a given project. She shows, that in equilibrium the agent might voluntarily abstain from gathering this information with positive probability. The intuition is here, that mixing strategies whether or not to acquire information he introduces asymmetric information on the part of the principal. This changes the optimal contract in favor of the agent. There are other papers coming to similar findings, that staying ignorant can be beneficial. However, our paper reveals a different channel through which the asymmetry can favor the uninformed player. In the simplest case, being ignorant allows her to use the theoretical chance of facing a strong opponent to gain against the weak, and vice versa. In a more recent article Schmitz (2007) analyzes optimal selling strategies when some buyers have hard information about their type and others are ignorant. He shows that the existence of ignorant types benefits the informed types in equilibrium. This is so because the presence of ignorant buyers makes it impossible for the seller to extract the total rent from informed buyers, since they always can mimic to be ignorant, too.

6 Uninformative or noisy signal

In this section we solve the model when $\epsilon \in (0, 1/2]$ and therefore player 1's first stage action is not perfectly observable, i.e. the signal is noisy ($\epsilon \in (0, 1/2)$) or totally uninformative ($\epsilon = 1/2$). The structure of this game can be seen in panel b.) of figure 3. Proposition 4 summarizes the results in this game.

Proposition 4. *If $\epsilon \in (0, 1/2]$ there is a unique perfect Bayesian equilibrium in pure strategies, in which player 1 spies in stage 1, player 2 "ignores" the signal in stage 2 and holds the belief $\beta(\sigma) = (s, s)$, and both players choose equilibrium efforts as in the full-information game (see equation (3)). Therefore, the equilibrium is given by*

$$\begin{aligned} A^{\epsilon>0} &= \left\{s, x_1^{\epsilon>0} = x_1^{NE}\right\} \\ B^{\epsilon>0} &= \left\{\beta(\sigma) = (s, s), x_2^{\epsilon>0}(\sigma) = (x_2^{NE}, x_2^{NE})\right\}. \end{aligned} \tag{14}$$

Moreover, if $\epsilon = 1/2$, this equilibrium is unique.

Proof. See appendix. □

Unfortunately, we cannot tell generally whether or not there are mixed-strategy equilibria when $\epsilon \in (0, 1/2)$. For $\epsilon = 1/2$ the pure-strategy equilibrium is the unique equilibrium of this game. As soon as the signal becomes imperfectly observable the staying-ignorant equilibrium vanishes. Player 1 has lost her advantage to commit to an effort level not on her ex-post best response function and is left worse off in certain cases.

As a consequence in this scenario both players choose their respective full information Nash equilibrium strategies. This is due to the fact, that player 1 now cannot use the strategic instrument "ignorance", because she cannot commit not to spy anymore. Therefore, though our model is different, we come to similar results as Bagwell (1995) or Morgan and Várdy (2007). Bagwell (1995) shows, that in sequential move strategic form two-player games all Stackelberg equilibria in pure strategies break down when the follower cannot observe the leaders strategy choice perfectly. The only remaining pure strategy equilibria are the simultaneous move Nash equilibria. However, in his paper there is still a mixed strategy equilibrium which converges to the Stackelberg equilibrium when the signal gets clearer. In a follow-up paper van Damme and Hurkens (1997) show that this "almost Stackelberg" mixed strategy equilibrium is the unique equilibrium remaining after certain equilibrium selection criteria. Morgan and Várdy (2007) apply a similar framework to a sequential-move Tullock contest where the follower has to pay in order to observe the leaders choice. They find, that there is only a unique pure strategy equilibrium, in which both player chose their respective simultaneous move Nash equilibrium efforts. Our model differs form theirs in two aspects. First, the contest itself is a simultaneous move game. Second, while they assume the follower has the opportunity to learn the leader's stage 1 action, in our model the uninformed player can choose to learn the type of her opponent. Moreover, we have, in a way, two-stage commitment. First, depending on the quality of the signal, player 1 may be able to commit to a particular stage 1 action. Then, if commitment in the first stage is effective, this allows her to commit to some stage 3 action which is favorable for her. The effectiveness of the stage 1 commitment allows her to make another commitment in stage 3.

7 Application - A market entry game

Assume there is a monopolist firm producing for an advertising intense market, like the market for cars, but a potential rival is about to enter the market and break the monopoly. The monopolist does not know the unit production cost of the potential entrant, but knows that her costs are $c_E = c_L$ or $c_E = c_H$, with 50 percent probability respectively. The monopolist's unit costs are c_I.

To keep things as simple as possible we assume an iso-elastic market demand function with a price-elasticity of demand equal 2:

$$x = p^{-2}.$$

Each firm can serve a given share q_i of the market, which is determined before in an advertising contest as for example in Friedman (1958). As a monopolist a firm's share is $q = 1$. Therefore, a firm faces a demand $\tilde{x}_i = q_i x$. Then, each firms profit function is

$$\Pi_i(c_i, q_i) = \underbrace{\frac{1}{4c_i}}_{=v_i} q_i.$$

Profits are simply a fraction q_i of the monopoly profit (the case when $q_i = 1$), and therefore the prize in the contest is the monopoly profit, $v_i = 1/(4c_i)$, $i = I, E$. Given the profit function we can easily establish the expected utility in the advertising contest:

$$EU_i = q_i v_i - x_i = \frac{x_i}{x_1 + x_2} \frac{1}{4c_i} - x_i.$$

Now we can use corollary 1 to see whether we end up in a corner solution equilibrium or not, i.e. whether or not all types of the entrant will actively participate in the contest. Given the two-point distribution we get after some manipulations $c_I > \bar{c}_I = \frac{c_H - c_L}{2}$. If $c_I < \bar{c}_I$ the high cost type of the entrant would not compete actively and we get partial deterrence through advertising efforts.[11] Therefore if a firm can commit to staying ignorant about the type of her opponent she might be able to deter some types of opponents effectively. This mechanism seems non-standard and contrasts well known results of, among others, Schmalensee (1983) or Fudenberg and Tirole (1984). These authors find that in markets with price competition there is a negative relation between advertising efforts of incumbent firms and market entry. Advertising prior to entry is a means to "captivize" consumers. Therefore, the less advertising efforts a firm spends prior to entry, the larger is the remaining market to compete in and thus competition gets fiercer, what in turn deters entry.[12]

[11] Of course our example is quite simple and stylized. Nevertheless, if we assume, for example, a more general dynamic game where the incumbent finally learns the opponent's type conditional on entry, the effect might carry over qualitatively.

[12] Fudenberg and Tirole (1984) call this the "lean and hungry look" as opposed to the "fat-cat effect". The latter means firms engage extensively in advertising efforts in order to ensure themselves a large share of the market while decreasing prize competition in the remaining market, thus accommodating entry.

In the following we assume an interior solution exists. According to proposition 1 equilibrium efforts are then equal to

$$\begin{aligned} x_I^{BE} &= \left(\frac{v_1\, E[v_2^{-0.5}]}{v_1 E[v_2^{-1}]+1}\right)^2 = \frac{\left(\sqrt{c_H}+\sqrt{c_L}\right)^2}{4\,(2\,c_1+c_H+c_L)^2} \\ x_E^{BE} &= \sqrt{v_E\, x_I^{BE}} - x_I^{BE}. \end{aligned} \tag{15}$$

If, for example, $c_I = 4$, $c_L = 2$, and $c_H = 9$ we find $v_I = 1/16$, $v_H = 1/36$, $v_L = 1/8$, and the corresponding equilibrium efforts $x_I^{BE} = 0.0135$, $x_H^{BE} = 0.00587$, and $x_L^{BE} = 0.0276$. Equilibrium expected utility of the incumbent in the contest is $EU_I^{BE} = 0.01855$ and the incumbent dissipates some 21.6 percent of her valuation in the contest.

What would now happen if the incumbent would observably learn the cost structure of the entrant? First the game would turn into a full information game. From (3) we can easily calculate equilibrium efforts for both players:

$$x_i^{NE} = \frac{c_j}{4\,(c_i+c_j)^2}\ ,\ i \neq j.$$

If the entrant is the low cost type c_L equilibrium efforts are $x_I^{NE} = 0.0138$ and $x_L^{NE} = 0.0278$. We see, that both players expend more effort than in the Bayesian game. If the entrant is of the high cost type we get $x_I^{NE} = 0.0133$ and $x_L^{NE} = 0.00592$. Here the incumbent spends less than in the Bayesian game but the entrant spends more than in the Bayesian game. Therefore, the expected effect of acquiring information on costs is not clear from the perspective of the incumbent, since with 50 percent probability cost rise or sink. But he knows that the entrant's expected effort will rise for sure in the full information game. The reason is, that in Tullock contests the favorites efforts are strategic complements to the underdogs, and the underdogs efforts are strategic substitutes to the favorites, as we described above. If we now compare equilibrium expected utilities we see that the incumbent player is actually better off without having full information. In our simple framework expected utility in the Nash game is given by

$$EU_I^{NE} = \frac{{c_2}^2}{4\,c_1\,(c_1+c_2)^2}.$$

Given the probabilities and concrete values for marginal production costs we find

$$E[EU_I^{NE}] = \frac{1}{2}\left(\frac{2^2}{16\,(4+2)^2} + \frac{9^2}{16\,(4+9)^2}\right) = 0.01845 < 0.01855 = EU_I^{BE}.$$

We see that indeed the incumbent is better off staying ignorant. This finding is in the simple case of a two point distribution true whenever the incumbent is the expected favorite of the game and his valuation is not too much larger than the one of his best possible opponent.[13]

8 Discussion: Signaling instead of Spying?

We showed that under certain circumstances it is beneficial for an uninformed player to stay uninformed in a contest. In our model the only way to resolve the informational asymmetry is the uninformed player's information acquisition or spying. However, in a conflict game like a Tullock contest, when one player benefits from ignorance, it might be the case that at least some types of the informed player suffer from this situation. Therefore, it seems intuitive that those types would have an incentive to resolve the informational asymmetry on their own. In principle we could assume many different channels through which such information transmission could happen.

Slantchev (2008) studies a crisis bargaining situation between two players, in which in a first stage player 1 makes a proposal how to split a given rent. If player 2 accepts, the game ends and both players get the proposed shares. If player 2 rejects both players compete in a Tullock contest for the rent. Principally, this contest is of two-sided asymmetric information, each player i is with probability p_i strong and otherwise weak. Quite interestingly, in equilibrium the behavior in stages 1 and 2 allows both players to infer at least partly how strong the opponent is, so that in the contest there is either full information or one-sided asymmetric information. Thus, the stage 1 and 2 actions are signals of the players that resolve the informational asymmetry at least partly. However, there are also "feigning" equilibria, in which a strong type demand only a small share in order to provoke a conflict, in which the opponent believes she is weak, therefore spending only low efforts in the contest and thus giving the actually strong player an advantage. For our paper Slantchev (2008) provides two nice insights. First, he shows how rational players behave in a conflict if there are informational asymmetries and there is no opportunity to spy but to signal. Second, his model nicely gives us a further application of a one-sided asymmetric information contest. In his model one-sided asymmetric information can emerge endogenously in a bargaining situation, when players are not able to peacefully agree on how to split the desired rent. Our model then explains behavior in the subgame starting after disagreement, if the uninformed player can

[13] A player is the expected favorite when her probability to win in the Bayesian game is larger than 50 percent.

acquire information himself. Therefore his paper is a close substitute but also a complement to ours.

Another recent article studying information transmission is Katsenos (2008). In his paper two players compete for a given rent with common value v. Players are heterogeneous in their effort costs and the information structure is two-sided asymmetric information. Specifically, each player is with probability ρ strong, i.e. her costs are low, or with probability $1 - \rho$ weak. Before the contest starts each player has the opportunity to send a signal, which he models as (unproductive) efforts spend already before the contest begins. In the second stage players spend effort in order to win the contest. In this framework he characterizes under which conditions sorting equilibria exist and thus the informational asymmetry can be overcome. He finds that a (symmetric) separating equilibrium, where the strong players signal their type, exists only if the probability of facing a strong opponent ρ is sufficiently low. The intuition here is that signalling strength against a strong opponent will make the competition fiercer and more wasteful. On the other hand a weak opponent will become discouraged by knowing he faces a strong opponent and the contest becomes less wasteful. This finding is interesting in that it shows that the actual holder of the information will only sometimes be able to credibly reveal it, leaving room for our analysis of information acquisition on the part of the uninformed player.

9 Conclusion

In this paper we analyzed Tullock contests with one sided asymmetric information. First we generally studied the Bayesian equilibria of those games. As in ordinary Tullock contests with full information, players' efforts are strictly increasing in their own valuation of the prize. However, in contrast to standard contests, we have to deal with corner solutions even in simultaneous play. This is due to the fact that the uninformed player sometimes chooses an equilibrium effort larger than the valuation of the informed player. Then we go on showing that even if the uninformed player has the opportunity to (costlessly) acquire the as yet unknown information about the other player's type, she might not have an incentive to do so. The reason is that ignorance is an important strategic instrument in the contest, enabling her to commit to an effort level that is almost surely not on her best response correspondence. Sometimes this effort level will discourage both weak and strong opponents due to the non-monotonicity of reaction functions. When information acquisition is only noisily observable this commitment opportunity is taken away from the uninformed player. In fact, if she could make information acquisition costly for her, she would do so under some circumstances in order to restore her commitment opportunity. Put differently, ignorance might have a value, depending on the distribution of valuations and the quality of the signal.

What are the implications of our analysis? First, the probability of conflict is reduced through the asymmetry in information. If we let the assumption of interior solutions in the contest we come to cases where the ignorance of the incumbent might be a means to completely deter some types of entrants. For example, if in the simple example above $c_H = 11$, keeping everything else equal, the high cost firm will never enter. Therefore in contests we should expect to see players that are not too different. Second, due to the preserved asymmetric information in the no-spying equilibrium the sum of efforts in the contest might be lower than in the full information case. In settings in which contest efforts are considered to be socially wasteful it might thus be desirable from a policy perspective to preserve the asymmetry (see also Hurley and Shogren (1998a), proposition 3).

A logical extension of our paper would be to allow for more general cost functions and contest technologies. In fact, we checked some simple examples with different values for Tullock's r, the discriminatory power of the contest, and our findings were qualitatively the same. Our intuition is, that our results might qualitatively carry over to all cases where the players's best response functions in the simultaneous full information game are hump-shaped. Another possibility is to extend the analysis towards a n-player contest. Probably the most interesting extension is to analyze the equilibrium information structure in a contest with two-sided asymmetric information, where players have both the opportunity to signal and spy. For this purpose our paper as well as the papers of Katsenos (2008) and Slantchev (2008) might be valuable starting points.

Appendix

A Proof of Proposition 1

In order to prove the proposition we first establish the following lemmata:

Lemma 1. *Player 1's best-response correspondence in the Bayesian game exists and is single-valued.*

Proof. This follows directly from the strict concavity of EU_1 $\left(\frac{\partial EU_1}{\partial x_1} > 0, \frac{\partial^2 EU_1}{\partial x_1^2} < 0 \; \forall x_2 > 0\right)$.
□

Lemma 2. *Independent of the information structure, player 1 will in every equilibrium play a pure strategy.*

Proof. To prove this we have to differentiate between four cases.

1. *Player 1 knows player 2's type and player 2 chooses a pure strategy in equilibrium.* In this case we have a full information game and it is well known that in the unique Nash equilibrium both players chose pure strategies (see Konrad (2009)).

2. *Player 1 does not know player 2's type, and all types of player 2 play a pure strategy in equilibrium.* In this case we know from lemma 1, that player 1 has a single best response, and accordingly he chooses a pure strategy in equilibrium.

3. *Player 1 knows player 2's type and player 2 chooses some mixed strategy $F(x_2)$ in equilibrium.* It is easy to show, that player 1's optimization problem in this case is isomorphic to the problem in the Bayesian game. Her expected utility is given by

$$EU_1^{Mix} = v_1 \int_a^b \frac{x_1}{x_1 + x_2} dF(x_2) - x_1$$

   By a simple change of variables ($x_2 = x_2(v_2)$ and $F(x_2) = G(v_2)$) we can transform this into the expected utility equation in the Bayesian game, since $x_2(v_2)$ is strictly monotonically increasing in v_2. As before we can use lemma 1 to prove player 1 plays a pure strategy.

4. *Player 1 does not know player 2's type, and all (some) types of player 2 play a mixed strategy in equilibrium.* We already saw that player 1 does not care whether player 2 randomizes or whether she does not know player 2's type. Not knowing the type is strategically similar to a randomization, since in the one case a given player randomizes her pure strategies and in the other case nature randomizes which type chooses a particular pure strategy. If unknown types play a mixed strategy this alters only the distribution of pure strategies chosen by known types, compared to the situation with known types playing a mixed strategy. Therefore, once again we can use lemma 1 to prove that player 1 will play a pure strategy in equilibrium.

□

Now we are able to prove the proposition. Player 1's FOC is given by

$$v_1 \int_a^b \left[\frac{x_2(v_2)}{(x_1^{BE} + x_2(v_2))^2} \right] dG(v_2) - 1 \stackrel{!}{=} 0.$$

Using player 2's best response function $x_2(x_1) = \sqrt{v_2\, x_1} - x_1$ we can simplify to get

$$\begin{aligned} 1 &= v_1 \int_a^b \left[\frac{\sqrt{v_2\, x_1^{BE}} - x_1^{BE}}{v_2\, x_1^{BE}} \right] dG(v_2) \\ \Leftrightarrow 1 &= v_1 \int_a^b \left[\frac{1}{\sqrt{v_2\, x_1^{BE}}} - \frac{1}{v_2} \right] dG(v_2). \end{aligned}$$

Using some further algebra we find the equilibrium effort level of player 1.

$$\begin{aligned} 1 &= v_1 \int_a^b \left[\frac{1}{\sqrt{v_2\, x_1^{BE}}} - \frac{1}{v_2} \right] dG(v_2) \\ \Leftrightarrow 1 &= v_1 \int_a^b \left[\frac{1}{\sqrt{v_2\, x_1^{BE}}} \right] dG(v_2) - v_1 \int_a^b \left[\frac{1}{v_2} \right] dG(v_2) \\ \Leftrightarrow 1 &= \frac{v_1}{\sqrt{x_1^{BE}}} \int_a^b \left[\frac{1}{\sqrt{v_2}} \right] dG(v_2) - v_1 \int_a^b \left[\frac{1}{v_2} \right] dG(v_2) \\ \Leftrightarrow x_1^{BE} &= \left(\frac{\int_a^b v_2^{-0.5}\, dG(v_2)}{\int_a^b v_2^{-1}\, dG(v_2) + v_1^{-1}} \right)^2 > 0 \end{aligned}$$

Player 2's equilibrium effort follows trivially from (2). Because the second order conditions for both players are weakly negative for all $x_1, x_2 \geq 0$ the efforts are actually optimal choices.

From lemma 2 we know that player 1 plays a pure strategy in any equilibrium. Thus, we do not need to check for mixed-strategy equilibria. Since player 2's best response is also single-valued (see 2) this implies a unique Bayesian Nash equilibrium in pure strategies, provided an equilibrium exists. Therefore the proof is complete. □

B Proof of Proposition 2

If the condition from corollary 1 is not fulfilled, the game has a corner solution. In this equilibrium there is a marginal type 2 ($\bar{v}_2 \in (a, b]$) who stays just passive, but all types with a higher valuation $v_2 > \bar{v}_2$ will be active. In such an equilibrium player 1 chooses her effort to maximize

$$EU_1^c = v_1 \left(\int_a^{\bar{v}_2} dG(z) + \int_{\bar{v}_2}^b \frac{x_1}{x_1 + x_2(v_2)} dG(z) \right) - x_1$$

If she chooses zero effort she gets zero utility. The corresponding FOC is given by

$$\frac{\partial EU_1^c}{\partial x_1} = v_1 \int_{\bar{v}_2}^b \left[\frac{x_2(v_2)}{(x_1^c + x_2(v_2))^2} \right] dG(v_2) - 1 \stackrel{!}{=} 0.$$

This is exactly the same as in proposition 1 except for the different boundaries of integration. Thus, the proof of player 1's corner solution effort follows immediately from the proof of proposition 1.

The effort for player 2 is still given by her best response function in (2). The marginal type of player 2 stays just passive in equilibrium. Put differently, $\sqrt{\bar{v}_2\, x_1^c} - x_1^c = 0 \Leftrightarrow \bar{v}_2 = x_1^c$, since v. Using (9) we get

$$\bar{v}_2 = x_1^c = \left(\frac{\int_{\bar{v}_2}^b z^{-0.5} dG(z)}{\int_{\bar{v}_2}^b z^{-1} dG(z) + v_1^{-1}} \right)^2.$$

The type of player 2 for whom this equation is fulfilled is the marginal player. According to (2) all types of player 2 with $v_2 > \bar{v}_2$ will spend positive effort $\sqrt{v_2\, x_1^c} - x_1^c$. All other types stay passive and spend zero effort.

As in proposition 1, uniqueness follows from (2), lemma 1, and lemma 2. □

C Proof of Fact 4

Let $G(v_2)$ be a uniform distribution on $[a, b] \subset \mathbb{R}^+$, such that the density is constant and $g(v_2) = \frac{1}{(b-a)}$ for $v_2 \in [a, b]$ and zero else. Using (7) and (8) we can calculate equilibrium efforts and utilities in the Bayesian game:

$$\begin{aligned} x_1^{uni} &= \frac{4\, v_1^2 \left(\sqrt{b} - \sqrt{a}\right)^2}{\left(v_1 \ln[\frac{b}{a}] + b - a\right)^2} \\ EU_1^{uni} &= \frac{4 v_1^3 \left(\sqrt{b} - \sqrt{a}\right)^2 (\ln[b] - \ln[a])}{(b-a)(v_1(\ln[b] - \ln[a]) + b - a)^2}. \end{aligned}$$

In this equilibrium player 1's effort and utility are strictly greater than zero. Her optimal strategy after spying is given in (3) and her expected utility from spying is

$$EU_1^{spy,uni} = \frac{v_1^3}{(b-a)}\left(\frac{1}{v_1+a} - \frac{1}{v_1+b}\right)$$

Now, player 1 is better off in the Bayesian game *without* spying whenever

$$D = \frac{1}{v_1+a} - \frac{1}{v_1+b} - \frac{4\left(\sqrt{b}-\sqrt{a}\right)^2(\ln[b]-\ln[a])}{(v_1(\ln[b]-\ln[a])+b-a)^2} < 0$$

or, equivalently, when player 1's valuation is intermediate, $\bar{v}_1 < v_1 < \hat{v}_1$, where the lower and upper bound are given by

$$\bar{v}_1 = \frac{\left(\sqrt{b}-\sqrt{a}\right)^3 \ln\left[\frac{b}{a}\right] - 2\sqrt{(b-a)\,\ln\left[\frac{b}{a}\right]\,\left(a-b+\sqrt{a}\sqrt{b}\,\ln\left[\frac{b}{a}\right]\right)^2}}{\ln\left[\frac{b}{a}\right]\left(4\sqrt{a}-4\sqrt{b}+\left(\sqrt{a}+\sqrt{b}\right)\ln\left[\frac{b}{a}\right]\right)}$$

and

$$\hat{v}_1 = \frac{\left(\sqrt{b}-\sqrt{a}\right)^3 \ln\left[\frac{b}{a}\right] + 2\sqrt{(b-a)\,\ln\left[\frac{b}{a}\right]\,\left(a-b+\sqrt{a}\sqrt{b}\,\ln\left[\frac{b}{a}\right]\right)^2}}{\ln\left[\frac{b}{a}\right]\left(4\sqrt{a}-4\sqrt{b}+\left(\sqrt{a}+\sqrt{b}\right)\ln\left[\frac{b}{a}\right]\right)}.$$

From inspection of $\hat{v}_1$ we see that this boundary is positive whenever the denominator is positive, i.e. $4\sqrt{a}-4\sqrt{b}+\left(\sqrt{a}+\sqrt{b}\right)\ln\left[\frac{b}{a}\right] > 0$. This is always the case when $b > a$. To see this suppose $b = ta$ for some scaling parameter $t > 1$. Then we get the following

$$\begin{aligned}4\sqrt{a}-4\sqrt{b}+\left(\sqrt{a}+\sqrt{b}\right)\ln\left[\frac{b}{a}\right] &= [4\sqrt{a}-4\sqrt{ta}+\left(\sqrt{a}+\sqrt{ta}\right)\ln[t] \\ &= \sqrt{a}\left(4-4\sqrt{t}+\ln[t]+\sqrt{t}\ln[t]\right).\end{aligned}$$

It is sufficient to have the term in brackets to be positive. By isolating the logartithms we get then

$$\begin{aligned}4-\sqrt{t}+\ln[t]+\sqrt{t}\ln[t] &> 0 \\ \Leftrightarrow 4(1-\sqrt{t})+\ln[t](1+\sqrt{t}) &> 0 \\ \Leftrightarrow \frac{4(1-\sqrt{t})}{(1+\sqrt{t})}+\ln[t] &> 0 \\ \Leftrightarrow \frac{4(\sqrt{t}-1)}{(\sqrt{t}+1)} &< \ln[t].\end{aligned}$$

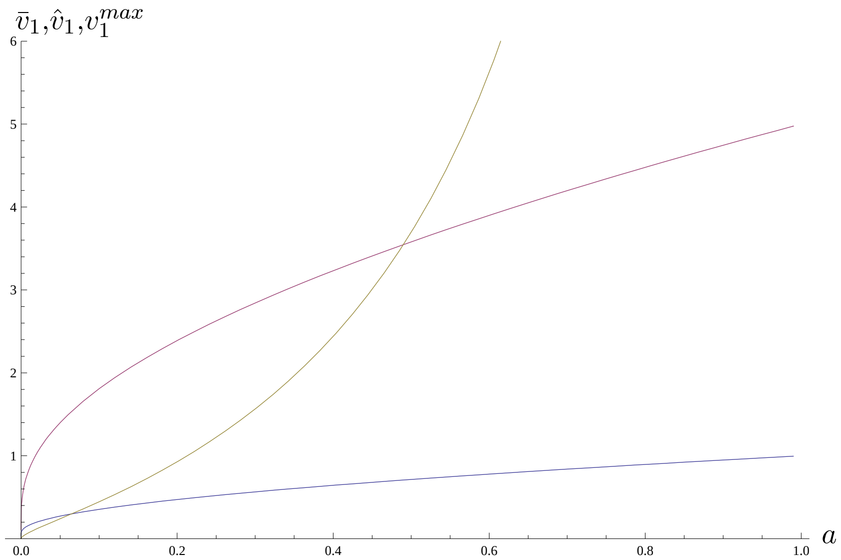

Figure 4: $\bar{v}_1$ *(lower graph) and* $\hat{v}_1$ *(upper graph) as a function of* a*. The parabola shows the maximum value of* v_1 *(*v_1^{max}*) for which we end up in an interior solution.*

In the limit case we have $t = 1$. Then the LHS and the RHS are equal (and zero). Therefore, to show that the RHS is strictly larger than the LHS it is sufficient to have the derivative of the LHS to be strictly smaller than the derivative on the RHS. Taking derivatives with respect to t we get

$$\begin{aligned} \frac{dLHS}{dt} &= \frac{4}{(\sqrt{t}+1)^2\sqrt{t}} \\ \frac{dRHS}{dt} &= \frac{1}{t}. \end{aligned}$$

So a sufficient condition for the upper boundary to be positive is

$$\begin{aligned} \frac{4}{(\sqrt{t}+1)^2\sqrt{t}} &< \frac{1}{t} \\ \Leftrightarrow t > 1 &\quad \vee \quad t \in (0,1). \end{aligned}$$

Since we assume $t > 1$ this is always fulfilled and therefore the upper boundary is always positive, implying that for all possible uniform distribution on the positive real numbers there are types of player 1 who is better off ignorant. □

In figure 4 we plotted $\bar{v}_1$ and $\hat{v}_1$ for $b = 1$. Spying is optimal only when the uninformed player is very weak *or* very strong, something that is generally true (see fact 5). Figure 4 shows that the higher a gets, i.e. the stronger player 2 is in expectation, the higher get both $\hat{v}_1$ and $\bar{v}_1$.

However, $\bar{v}_1$ is growing faster and thus the area where the player is better off without spying is increasing in a. Moreover, checking fact 3 we see that when $\int_a^b z^{-1} dG(z) = 1/v_1 \Leftrightarrow v_1 = \frac{(a-1)}{\ln[a]}$ it is always, i.e. for all a, optimal not to spy. Put differently, $\bar{v}_1 < \frac{(a-1)}{\ln[a]} < \hat{v}_1$. When a increases $\frac{(a-1)}{\ln[a]}$ converges to $\bar{v}_1$ from above.

D Proof of Fact 5

We first consider the case $v_1 \to 0$. D is given by

$$D = v_1^3 \left(E\left[\frac{1}{(v_1+v_2)^2}\right] - \frac{E\left[v_2^{-1}\right] E\left[v_2^{-0.5}\right]^2}{\left(v_1 E\left[v_2^{-1}\right]+1\right)^2} \right),$$

where we change notation from integrals to expectation operators in order to make the exposition clearer. The expectation is always taken with respect to v_2. As $v_1 \to 0$, $D \to 0$ as can be seen from the expression for D. To determine from which side D approaches zero we examine the sign of the expression in brackets,

$$E\left[\frac{1}{(v_1+v_2)^2}\right] - \frac{E\left[v_2^{-1}\right] E\left[v_2^{-0.5}\right]^2}{\left(v_1 E\left[v_2^{-1}\right]+1\right)^2}.$$

Let us set $v_1 = 0$ to determine the sign of the expression in brackets.

$$E\left[\frac{1}{v_2^2}\right] - E\left[\frac{1}{v_2}\right] E\left[\frac{1}{\sqrt{v_2}}\right]^2$$

We use Jensen's inequality twice to determine the sign:

$$\begin{aligned} E\left[\frac{1}{v_2^2}\right] - E\left[\frac{1}{v_2}\right] E\left[\frac{1}{\sqrt{v_2}}\right]^2 &> E\left[\frac{1}{v_2^2}\right] - E\left[\frac{1}{v_2}\right] E\left[\frac{1}{v_2}\right] \\ &> E\left[\frac{1}{v_2^2}\right] - E\left[\frac{1}{v_2}\right]^2 \\ &> E\left[\frac{1}{v_2^2}\right] - E\left[\frac{1}{v_2^2}\right] = 0. \end{aligned}$$

As we can see D is positive when $v_1 \to 0$.

Now we turn to $v_1 \to \infty$. Again we only need to look at the expression in brackets. If we take the limit to infinity only the highest order terms of v_1 have to be considered. Hence our expression reduces to:

$$\frac{1}{v_1^2} - \frac{E\left[\frac{1}{v_2}\right] E\left[\frac{1}{\sqrt{v_2}}\right]^2}{v_1^2 E\left[\frac{1}{v_2}\right]^2} = \frac{1}{v_1^2} - \frac{E\left[\frac{1}{\sqrt{v_2}}\right]^2}{v_1^2 E\left[\frac{1}{v_2}\right]} = \frac{1}{v_1^2}\left(1 - \frac{E\left[\frac{1}{\sqrt{v_2}}\right]^2}{E\left[\frac{1}{v_2}\right]}\right).$$

Again we make use of Jensen's inequality:

$$1 - \frac{E[\frac{1}{\sqrt{v_2}}]^2}{E[\frac{1}{v_2}]} > 1 - \frac{E[\frac{1}{v_2}]}{E[\frac{1}{v_2}]} = 0.$$

We see, that $D > 0$ when $v_1 \to \infty$. This completes the proof. □

E Proof of Proposition 3

The proof that this is indeed an equilibrium follows from the discussion in the text. Uniqueness follows from uniqueness of the equilibria in the subgames following the first stage decision and the assumption that for $D = 0$ player 1 will choose to acquire information. Hence, player 1 will never mix between s and n in stage 1. □

F Proof of Proposition 4

Given $A = \{s, x_1^{NE}\}$, by consistency of beliefs player 2 will always believe that player 1 spies regardless of her signal and it is optimal for her to play x_2^{NE}. On the other hand, given $B = \{\beta(\sigma) = (s,s), x_2(\sigma) = (x_2^{NE}, x_2^{NE})\}$ player 1 will find it optimal to spy and play her best response x_1^{NE}. We prove uniqueness (in pure strategies) by contradiction. Assume that player 1 does not acquire information. By consistency of beliefs player 2 will believe that player 1 did not spy regardless of her signal. Hence she will play x_2^{BE}. In this case player 1 will find it profitable to deviate and acquire information as player 2 will not spot the deviation and react on it. □

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