Product Heterogeneity, Trade, and the Dynamics of Industry*

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Abstract

Entry and exit of firms into industry depends on profitability rather than on productivity differences. This paper analyzes the dynamic impact of trade liberalization when product heterogeneity, rather than heterogeneity in physical productivity, determines profitability and export selection. I develop a model featuring goods of heterogeneous attributes and heterogeneous consumer demand for these attributes. In equilibrium, consumers and goods tend to be matched assortatively, i.e., high valuation consumers account for a relatively large share of the sales for high attribute firms. Foreign market access is subject to fixed costs and the possibility to export favors more profitable firms. The model's central new prediction is that the import competition tends to hit domestic exporters more than non-exporters since foreign and domestic exporters produce similar goods and hence compete for similar customers. It is, therefore, not certain that exposure to trade induces the composition of firms to shift towards the ex ante more profitable entities.

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1 Introduction

The literature using micro data at the establishment level consistently finds that within-industry firm exit and entry is one of the driving forces of aggregate industry dynamics. Theoretical models of Hopenhayn (1992) and Melitz (2003), among many others, have given particular prominence to the heterogeneity in physical productivity as the driving force behind firm turnover. Firm's exit and entry decisions are, however, based on profits, rather than on productivity.

For example, while the case can easily be made that General Motors, Toyota, and BMW work at different productivity levels, a non-negligible feature of their heterogeneous success is the difference in the cars these firms produce. These cars are produced with emphasis on different aspects – in this example with emphasis on either size, mileage, or top speed – and they use inputs of different costs. Most importantly, these cars are also sold to different sets of customers, i.e., they are heterogeneous in the demand they face.[1]

Also in the aggregate, heterogeneity in physical productivity is not the dominant force of industry selection. In a recent study on reallocation and firm turnover in the US, Foster, Haltiwanger, and Syverson (2008) find that even in industries with highly standardized output such as concrete, demand heterogeneity accounts for at least three times as much industry reallocation as does heterogeneity in physical productivity.

How does trade shape the evolution of industry when differences in products and in the demand that these products face, rather than physical productivity differences, are the source of firm heterogeneity? The theoretical literature is well aware that differences in product quality, in the location of production, and in other product attributes may be the underlying force of the heterogeneity in profits,[2] but it has so far largely disregarded the impact of this fact on industry dynamics based on the assertion that any form of firm heterogeneity is equivalent to heterogeneity in physical productivity. In this paper, I document that this is not the case.

In the existing literature, goods are consumed by a representative agent who maximizes the effective consumed quantity, i.e., higher good quality and lower per unit cost are isomorphic. The key novel ingredients of the model developed in this paper is the existence of heterogenous consumers that differ in their tastes for product attributes. Firms are heterogeneous in the

[1] For example, the recent spike in oil prices rose has strongly decreased sale's of General Motors large trucks, did not substantially affect BMW's sales, yet actually led to an increase in the sales of many of Toyota's models

[2] Indeed, Baldwin and Harrigan (2007) note that exported goods tend to be of higher unit value than domestic goods and, therefore, argue that the Melitz (2003) model should be slightly rewritten into a version where firms are heterogeneous in their quality, high quality firms are more profitable and select into exporting.

attributes of their products. In equilibrium, the group of consumers that have a strong preference for the attributes of a given firm account for a relatively large share of this firm's revenue.

The key result of this preference structure is that a firm's sales decrease more when a new firm with a similar product enters the industry than when a firm with a very dissimilar product enters. Thus, import competition hits firms with different output differently and does not necessarily crowd out firms that were least profitable before markets were opened to trade. In contrast, in models of productivity heterogeneity, due to the assumed preference structure, increasing import competition hits all firms equally and is thus likely to crowd out the least productive firms that were characterized by low profits before markets are opened to trade.

The first contribution of this paper is to develop a model that introduces the notion of within-industry market segmentation into the standard love-of-variety setup of Krugman (1980). Following Mussa and Rosen (1977) and Auer and Chaney (2008a and 2008b) preferences feature both heterogeneous good attributes and consumers that differ in their taste for product attributes. An "attribute" is any product characteristic, such as its color, the language of its user manual, or its quality. Firms sell their output to heterogeneous consumers. Consumers are heterogenous in the valuation for attributes and also face an idiosyncratic consumer-firm specific taste shock. Due to the presence of the latter idiosyncratic taste shock, some consumers end up buying goods for which attributes the have a dislike, but on average they buy most from firms which best fit their attribute preferences.

The second contribution of the paper is show how trade affects intra-industry reallocation in the dynamic model of industry of Melitz (2003) augmented by the before-mentioned preference structure. The central new result is to show that since firms suffer more from entry of similar competitors, it is not always true that trade induces a shift of the distribution of firms towards the more profitable ones since precisely those firms that find it profitable to export will also experience the strongest increase in competition, because also foreign import competition tends to produce similar goods.

I first highlight the effect of opening markets to trade in a simple economy featuring two types of firms and two types of consumers. Assume that trade leads to high-attribute firms selecting into the export sector. Undoubtedly, this has a benefit for high-attribute producers, who at least break even when exporting. However, if countries are symmetric, also foreign high-attribute producers export, which tends to hurt domestic high-attribute producers much more than low-attribute prodcuers. The net effect on the relative profits of exporters and non-exporters thus depends on the net gains from trade and the difference in how different firms are affected by competition from high-attribute firms. Either effect can dominate. For example, the relative profit of the non-exporters increases with trade when the fixed cost of trading is sufficiently high such that exporters barley break even on exporting. Second, I extend these results to a preference structure featuring a continous distribution of product attributes.

2 Why Attributes and Demand Heterogeneity Matter

It is obvious that firms face heterogeneous demand, but the underlying forces that determine this heterogeneity are less understoud. This paper emphasizes the micro foundations of demand heterogeneity, which is essential to the understanding of the dynamics of industry under trade.

A first ingredient of the model is that goods in an industry are generally not the same, i.e. they differ in product characteristics or "attributes." The second characteristic is that also consumers are not the same, i.e. they differ in their valuation for attributes. It is the interplay of these two ingredients that leads to endogenous market segmentation.

In general, consumers are not homogenous in their valuations for attributes. Among many other dimensions, consumers differ in their taste, in their willingness to pay for quality, and also in their location. A Ferrari isn't sold to the same group of customers as a Fiat, Apple's Mac computer targets consumers that are quite different from the market for Windows based PCs, and even the most successful New York Pizza shops face very little demand from Chicago residents.

In the model developed below, firms sell a good of heterogeneous attributes to consumers with heterogeneous valuation (or dislike) for the attribute. This attribute may be good quality, heterogeneity in location as in the standard Hotelling model, or simply a subjective attribute such as the color of the good. Among others this two-sided heterogeneity has been emphasized by:

- De Loecker (2008) estimates that properly accounting for product attributes leads to productivity gains that are only half as big as without accounting for product attributes.

- Foster, Haltiwanger, and Syverson (2008) highlight the importance of demand heterogeneity.

- Hallak (2006 and 2007) quality differentiation matters for trade flows.

- Hallak and Schott (2008) estimate cross country differences in product quality and find that this is enormous.

- P.Goldberg and Verboven (2001) and Auer and Chaney (2008b) highlight the importance of quality differentiation in the car industry. Similarly, the entire empirical literature building on

Barry, Levinson, and Pakes (1995) focuses on attribute differentiation.

3 A model of within industry segmentation

3.1 Demand

In this section, I develop an economy in which all firms produce good that differs in its attributes a. Differences in attributes can be seen as differences in good quality, but also subjective differences in the characteristics of a good, for example in its design. Consumers are heterogenous in two dimensions. First, they are characterized by their love of the attribute a. Second, they are also subject to a good-consumer specific taste shock. The latter shock reflects idiosyncratic aspect of a consumer likes a given product. For example, while all consumers value a Toyota's quality, some dislike the image of the brand.

The world is populated by n identical countries each with a population of L who can consume a differentiated good. In each country there are many firms which produce the latter good at different levels of the attribute. Throughout the analysis, let $i \epsilon I$ index consumers (individuals) and $j \epsilon J$ index firms. A firm j is endowed with the production technology to produce a good of quality $a = a_j$. A consumer i is endowed with income w, a valuation draw $v_i \geq 0$, and a consumer-good specific draw $x_{i,j}$ for each firm in J. Consumers only care about the quality and valuation-adjusted effective quantity of the industry's good. Denoting the quantity consumer i buys from firm j by $q_{i,j}$, a consumer's utility U_i is given by

$$U_i = \sum_{j \epsilon J} q_{i,j} e^{a_j v_i + x_{i,j}} \tag{1}$$

Subject to non-negativity for each pair i, j $q_{i,j} \geq 0$ and to her budget constraint.

$$\sum_{j \epsilon J} q_{i,j} p_j \leq w \tag{2}$$

This formulation of utility (1) implies that consumers care only about the effective amount of a good consumed and therefore all goods are perfect substitutes to them. However, different consumers have different rates of substitution between different varieties and in equilibrium, certain types of consumers are more or less likely to buy certain types of a goods. For expositional clarity, I assume that there are only two types of consumers. A fraction π_v of the total population of L is of high type h with valuation v_h, while the rest of the other population $((1 - \pi_v) L)$ is of low type l with valuation v_l, where $v_h > v_l$ holds. Note that when $v_l > 0$, a all consumers value higher attribute goods and one can speak of "quality" as in Auer and Chaney (2008a and 2008b). In this paper, I however do not necessarily assume that $v_l > 0$, so that the analysis also extends to product characteristics other than quality.

Consider first only the term $e^{a_j v_i}$ in (1). The key feature of this term in the preferences (1) is that the rate at which consumers value (or dislike) the attribute differs between consumers with different v_i. Assume that two consumers of valuations v_l and v_h are offered to buy a certain good a_l at price p_l or a good a_h at price p_h where $a_h > a_l$. What is the maximum price difference between p_l and p_h at which each consumer would prefer the high a good? For consumer h, this would be price ratio $p_h/p_l = e^{v_h(a_h - a_l)}$. However, consumer l would only be willing to buy only at a relative price up to $p_h/p_l = e^{v_l(a_h - a_l)}$. Because higher valuation consumers value the attribute more, in equilibrium they constitute the relatively larger group of consumers of these goods.

Next, consider only the term $e^{x_{i,j}}$ in (1). $x_{i,j}$ is a consumer-firm specific shock, reflecting the fact that some consumers like or dislike not only general attributes, but also have preferences for the output of a specific firm. Anderson (1982) and in particular Gabaix et al. (2006b) show that this specification yields a ideal variety-microfoundation for CES demand. In (1), the idiosyncratic taste shock introduces market power to the model: although firms cannot observe $x_{i,j}$, they can engage in first degree price discrimination by charging a higher price and only attracting consumers with high $x_{i,j}$ draws. Throughout the analysis, I assume that $x_{i,j}$ is distributed (maximum) Gumbel with scale and shape parameters 0 and $1/\beta$ respectively.

$$G_x(x_{i,j}) = \exp[-\exp[-x_{i,j}\beta]] \tag{3}$$

The closed form assumption on the consumer-firm specific taste shocks (3) is less restrictive that would seem at first place, since in equilibrium consumers buy only from the attribute-adjusted maximum realization of $x_{i,j}$ and the economy exhibits many firms. The distribution of this maxima converges to the Type I Extreme value function for a wide class of underlying distributions. In addition, Gabaix et al. (2006a) demonstrate that the maximum set is very comparable to the assumed function even for a small number of firms. Note also that consumer-firm specific shocks are orthogonal to firm quality or consumer valuation. Also, these shocks are orthogonal across firms and consumers: $x_{i,j} \perp x_{i,n}$ for $n \neq j$.

3.2 Production

Other than that there are differences in good attributes a rather than in productivity, the production setup is similar to Melitz (2003). In each symmetrical country there is only one factor of production, labor, which is in-elastically supplied at its aggregate level L. Labor is used to found new companies, to finance a flow-cost f to keep existing companies alive, and to produce each firm's output. Since all economic activities use domestic labor as an input, I normalize all prices and costs by the wage in the respective country.

In each country, firms are heterogenous in the characteristics of their output a_j. The industry is contested and entry is costly. Potential producers can enter the industry by paying a once and for all cost F to enter the market. Once a firm has paid this entry cost, the product characteristics a are randomly drawn from a distribution $G_a(a)$. $G_a(a)$ may have positive support over $(-\infty, \infty)$.

After entering the industry and before producing, each firm also has to pay a flow cost f in order to keep its production facility alive and serve its home market. I assume that any firm that leaves the home market and therefore does not pay f immediately goes out of business forever. In the open economy, serving the foreign market is optional and also subject to a flow cost t.

After entering the market, and potentially also after paying t to access the foreign market, the firm can produce and sell any quantity it desires at constant marginal cost. Denote the constant marginal cost to sell at home by c_j and the constant marginal cost to sell at the foreign market by c_j^*. The marginal cost of selling at home and abroad are equal to

$$c_j = e^{ca_j} \quad \text{and} \quad c_j^* = \tau e^{ca_j} \tag{4}$$

(4) allows for the possibility that the marginal cost of production differs across goods with different characteristics. Selling in the foreign market is subject to iceberg transportation costs and $\tau > 1$. The two markets are separated and consumers cannot arbitrage the good between home and foreign themselves. All firms are assumed to take the prices of other firms as a given, neglect their impact on the aggregate price index. Last, firms do not know the realizations of $x_{i,j}$ (or alternatively cannot price discriminate within a country).

4 Static Industry Equilibrium

At each point in time, consumers maximize (1) subject to their business constraint (2). A firm's strategy is to choose output, price in each market they are active, to decide whether to access the foreign market, and to decide whether to leave the market altogether. Potential foreign entrants can decide to enter the home market.

I denote a firm's price by p_j. Because consumers only care about the attribute- and valuation-adjusted effective quantity of the good they consume, they maximize $e^{a_j v_i + x_{i,j}}/p_j$. With the continuos distribution of consumer-firm taste shocks $x_{i,j}$, the probability of ties is 0, and the consumer hence only chooses one firm and then spends all his income on this firm's output. All firms are assumed to take the prices of other firms given and do not know the realizations of $x_{i,j}$. Thus, each firm faces the following expected demand at home.

$$D_j\left(a_j, p_j, v_j\right) = \sum_{i \epsilon I} \frac{w}{p_j} \int\limits_{x_{i,j} \epsilon X} g_x\left(x_{i,j}\right) \Pr\left(\frac{e^{a_j v_i + x_{i,j}}}{p_j} = \max_{n \epsilon J} \frac{e^{a_n v_i + x_{i,n}}}{p_n}\right) dx_{i,j} \tag{5}$$

Since there are infinitely many consumers i, expected and actual demand are identical, and I will hence speak simply about demand. There are four elements of demand (5). First, assume that a firm would know the realization of the consumer-firm specific taste shocks only for itself, but not for any of its competitors $n \neq j$. Then, the probability that a certain consumer would buy from firm j were equal to the probability that $e^{a_j v_i + x_{i,j}}/p_j$ is the equal to $\max_{n \epsilon J}\left[e^{a_n v_i + x_{n,j}}/p_n\right]$, i.e. a firm compares itself not to each of its competitors, but just to the maximal utility a consumers can expect from the rest of the market.

The second element of (5) is reflected by the inner integration over $x_{i,j}$. A firm does not know $x_{i,j}$ but it knows the distribution thereof and hence evaluates the expected probability that it is the best firm for consumer i. $\int\limits_{x_{i,j} \epsilon X} g_x\left(x_{i,j}\right) \Pr\left(\frac{e^{a_j v_i + x_{i,j}}}{p_j} = \max_{n \epsilon J} \frac{e^{a_n v_i + x_{i,n}}}{p_n}\right) da_{i,j} = E\left[\Pr\left(\frac{e^{a_j v_i + x_{i,j}}}{p_j} = \max_{n \epsilon J} \frac{e^{a_n v_i + x_{i,n}}}{p_n}\right)\right]$ is hence the expected probability that a firm sells to consumer i.

The third element in (5), $\frac{w}{p_j}$ simply reflects the fact that when a consumer buys from firm j, he will spend all his income w on the firm's output. Finally, the outer summation in (5) over i acknowledges that there are many consumers that may differ in their v_i.[3]

Since the consumer-firm specific taste shocks are independent, the probability that firm j is the maximum choice of i of all firms is equal to the product of all probabilities that the firm is a

[3]This formulation assumes that firms know valuation draws v_i. Because the demand from each v-type is exactly of the same shape, this is no different than if firms did not know v. In that case, demand would simply include the integration of the density over v draws.

better choice for consumer i than each single firm $n \neq j$.

$$\Pr\left(\frac{e^{a_j v_i + x_{i,j}}}{p_j} = \max_{n\epsilon J}\frac{e^{a_n v_i + x_{i,n}}}{p_n}\right) = \prod_{n\neq j}\Pr\left(\frac{e^{a_n v_i + x_{i,n}}}{p_n} < \frac{e^{a_j v_i + x_{i,j}}}{p_j}\right)$$

In Appendix 1, I solve this expression for using the Gumbel distribution of consumer-firm specific taste shocks (3) yielding

$$\prod_{n\neq j}\Pr\left(x_{i,j} < (a_j - a_n)\, v + \ln\left(\frac{p_n}{p_j}\right)\right) = \exp\left[-p_j^{1/\beta}\exp\left[-\beta\left(a_j v_i + x_{i,j}\right)\right]\sum_{n\neq j} p_n^{-\beta}\exp\left[v\beta q_n\right]\right] \tag{6}$$

Three facts are noteworthy in (6). Firm j's probability to sell to a given consumer is higher if p_j is lower or if other goods are more expensive. Similarly, for a given price and a positive v, the firm always tends to sell more if its attribute a_j is higher than the attribute of its competitor. The reverse holds true for $v < 0$. Most importantly, the latter two effects are more pronounced for larger v, i.e., consumers with very strong (positive or negative) valuation for attributes tend to put less weight on the idiosyncratic taste shocks and more on attributes. Because higher v_i consumers put more weight on attributes, in equilibrium, they are a relatively important group of customers for high a_j firms. That is, for a high a firm j many consumers with high valuations will find it optimal to buy from j even though $x_{i,j}$ is low. The same is not true for low valuation consumers, who in equilibrium tend to buy relatively more often from low valuation firms. I next solve for the total demand.

$$D_j\left(a_j, p_j\right) = \Gamma\left(1-\beta\right)\sum_{i\epsilon I}\frac{w}{p_j}\frac{p_j^{-(1+\beta)}\exp\left[\beta v_i a_j\right]}{\sum\limits_{n\epsilon J} p_n^{-\beta}\exp\left[\beta v_i a_n\right]} \tag{7}$$

$\Gamma\left(1-\beta\right)$ is the Gamma function. Despite the rich structure of preferences, demand (7) takes a simple functional form that is closely related to the common Love of Variety preference structure used in Krugman (1980), Meltiz (2003), and many other academic articles.

As is the case in the current literature, each firm faces a constant elasticity demand irrespective of the market environment. The elasticity of demand is equal to $-\left(1+\beta\right)$.[4] This total elasticity is the result of the effect of the intensive and extensive margin. Since a consumer that has decided to buy from firm j spends all her income on the firm's output, the average sales per customer are equal to $\frac{w}{p_j}$. In addition, the number of customers responds to the absolute price of the firms

[4] The ususal assumption that firms neglect the effect their pricing decision on the economy's pricing indexe(s) $\sum\limits_{n\epsilon J}\left(p_n^{-\beta}\exp\left[v\beta q_n\right]\right)$ is made.

output with an elasticity of $-\beta$ due to the shape of the consumer-firm taste shocks (3).[5] The total d demand elasticity is hence constant and equal to $-(1+\beta)$, leading to the firm's pricing decision to be independent of market environment and equal to $p_j = \left(\frac{1}{\beta}+1\right)e^{-ca_j}$. The demand structure also embodies the standard CES demand for the special case with homogenous a_n.

In contrast to the current literature, the price index – measuring thoughness of competition – is not the same for all consumers, but varies with v_i. The key mechanism captured in the demand function (7) is that firm's demand reacts stronger to entry of similar goods producers than it does when a firm with a dissimilar a enters the market. The next section develops this intuition further for a simple two attributes - two valuations example.

5 Entry and Endogenous Industry Segmentation

I next analyze a very simple version of the economy with two different qualities a_h and a_l, where $a_h > a_l$. This very simple structure is chosen to highlight the intuition, which are somewhat concealed in the full model that I develop in the next section.

A firm is of high type h with probability $\pi_a \epsilon [0,1]$ and of low type l with probability of $(1-\pi_a)$. In the equilibrium of this section, I shall assume that parameters are such that high a_h firms are more profitable in the closed economy, and also that parameters are such that low a_l firms just break even, which is possible for a range of parameters since a_l firms are indifferent between entering and exiting the industry, and in equilibrium, thus a fraction $s_{a_l} \epsilon [0,1]$ of the population chooses to stay and produce, while $1-s_{a_l}$ firms exit the industry.

The reason for assuming that parameters are in this region is the following. The parameter range were one kind of firms does not make a profit allows for an extensive margin, i.e., firms voluntarily leaving the industry. This assumption allows to relate the model to Melitz (2003). The central finding in Melitz (2003) is that trade liberalization induces the distribution of active firms to shift toward the ex-ante more profitable ones, i.e., high productivity firms. Using this very simple setup, I document that this need not necessarily be the case when product heterogeneity is the key driver of trade.

Throughout the paper, I also assume that

$$v_h > c > v_l$$

[5] Insterestingly, the fact that idiosyncratic taste shocks can lead to a CES type demand structure demonstrates how closely the Eaton and Kortum (2002) model and especially Bernanrd Eaton and Kortum (2004?) are related to the literature relying on the Dixit ind Stiglitz CES preferences.

, i.e., that some consumer prefers high a goods also when taking into account that they are more expensive, but other consumers either dislikes a or does not like them enough to justify the higher price. This is an important assumption, since with it, there is no uniformly "better" draw for a, but the relatvie profitablility of l and h firms depends on equilibrium entry.

The Timing of this simple economy is the following:

1. N entrants pay the fixed cost F and enter the industry.

2. Each entrant receives an $a-$draw. With probability π_a, the draw is equal to a_h and with probability $1-\pi_a$ it is equal to a_l.

3. Only a fraction s_{a_l} of the a_l firm choose to stay in the industry. The other $N(1-\pi_a)(1-s_{a_l})$ entrants leave the industry.

4. All h and the remaining l firms produce and pay a fixed cost f to keep their business alive.

5. Firms Produce and Sell.

5.1 Equilibrium in a Closed Economy

Normalizing $\Gamma(1-\beta)w=1$, demand has the shape

$$D(a_j) = p(a_j)^{-(1+\beta)} L\left(\pi_v \frac{\exp[\beta v_h a_j]}{NA_h} + (1-\pi_v)\frac{\exp[\beta v_l a_j]}{NA_l}\right)$$

Where A_l^{-1}, A_h^{-1} are the respective price indices for consumers v_l and v_h – the central measure of the thoughness of competition in this economy.

$$\begin{aligned}
A_h &\equiv \pi_a \exp[\beta(v_h-c)a_h] + s_{a_l}(1-\pi_a)\exp[\beta(v_h-c)a_l] \\
A_l &\equiv \pi_a \exp[\beta(v_l-c)a_h] + s_{a_l}(1-\pi_a)\exp[\beta(v_l-c)a_l]
\end{aligned}$$

Denote the revenue of l firms $\Pi(a_l)$ and the revenue of h firms by $\Pi(a_h)$.Since they face a constant elasticity of demand, and the marginal cost of production is equal to $\exp[\beta a_i]$, the revenue of firms is equal to

$$\Pi(a_l) = L\left(\pi_v \frac{\exp[\beta(v_h-c)a_l]}{NA_h} + (1-\pi_v)\frac{\exp[\beta(v_l-c)a_l]}{NA_l}\right) \quad (8)$$

$$\Pi(a_h) = L\left(\pi_v \frac{\exp[\beta(v_h-c)a_h]}{NA_h} + (1-\pi_v)\frac{\exp[\beta(v_l-c)a_h]}{NA_l}\right) \quad (9)$$

Where A_l, A_h are the respective price indices for consumers v_l and v_h - the central measure of the thoughness of competition in this economy.

$$\begin{aligned}\frac{A_h}{N} &= \pi_a \exp\left[\beta\left(v_h - c\right)a_h\right] + s_{a_l}\left(1-\pi_a\right)\exp\left[\beta\left(v_h - c\right)a_l\right] \\ \frac{A_l}{N} &= \pi_a \exp\left[\beta\left(v_l - c\right)a_h\right] + s_{a_l}\left(1-\pi_a\right)\exp\left[\beta\left(v_l - c\right)a_l\right]\end{aligned}$$

Given the previous assumptions, there are two conditions that determine the equilibrium. First, profits for l firms have to equal 0 for an interior s_{a_l}. Second, only h firm do make a positive profit, and the number of potential entrants N is hence determined by the probability of being of the h type and the net profits conditional on being of h type. Since profits equal a share $(1+\beta)^{-1}$ of revenue minus the fixed cost f, the equilibrium condition is

$$(1+\beta)^{-1}\Pi\left(a_h\right) > (1+\beta)^{-1}\Pi\left(a_l\right) = f$$

Because in this equilibrium only a_h firms make a positive profits, the number of entrants N depends only on the expected profits conditional on an a_l draw times the probability of receiving this draw.

$$F = \pi_a\left((1+\beta)^{-1}\Pi\left(a_h\right) - f\right)$$

In general equilibrium, the two conditions that a_l leave the industry, while all expected profits occur in for a_h draws pins down the ratio of profits: $\frac{\Pi(a_h)}{\Pi(a_l)} = \frac{\frac{F}{\pi_a}+f}{f}$. Leading to the equilibrium share of l firms staying in the industry of s_{a_l} that satisfies

$$\begin{aligned}\pi_v\frac{\exp\left[\beta\left(v_h-c\right)a_l\right]}{A_h} + \left(1-\pi_v\right)\frac{\exp\left[\beta\left(v_l-c\right)a_l\right]}{A_l} &= \frac{N}{L}(1+\beta)f \\ \pi_v\frac{\exp\left[\beta\left(v_h-c\right)a_h\right]}{A_h} + \left(1-\pi_v\right)\frac{\exp\left[\beta\left(v_l-c\right)a_h\right]}{A_l} &= \frac{N}{L}(1+\beta)\left(f+\frac{F}{\pi_a}\right)\end{aligned}$$

thus, s_{a_l}, the fraction of l firms that survives, is uniquely determined

$$s_{a_l} = \frac{\pi_a}{1-\pi_a}\frac{\left(1-\pi_v\right)\exp\left[\beta\left(v_h-c\right)a_h\right]\lambda_{v_l} - \pi_v\exp\left[\beta\left(v_l-c\right)a_h\right]\lambda_{v_h}}{\pi_v\exp\left[\beta\left(v_l-c\right)a_l\right]\lambda_{v_h} - \left(1-\pi_v\right)\exp\left[\beta\left(v_h-c\right)a_l\right]\lambda_{v_l}}$$

Where

$$\begin{aligned}\lambda_{v_h} &= \exp\left[\beta\left(v_h-c\right)a_h\right] - \exp\left[\beta\left(v_h-c\right)a_l\right]\left(1+\frac{F}{f\pi_a}\right) \\ \lambda_{v_l} &= \exp\left[\beta\left(v_l-c\right)a_l\right]\left(1+\frac{F}{f\pi_a}\right) - \exp\left[\beta\left(v_l-c\right)a_h\right]\end{aligned}$$

Since it is always true that $\lambda_{v_l} > 0$, for a positive s_{a_l}, we need $\exp\left[\beta\left(v_h - c\right)\left(a_h - a_l\right)\right] > \left(1 + \frac{F}{f\pi_a}\right)$. Thus, for a positive s_{a_l}, we need that the numerator and the denominator are of opposite signs.

$$s_{a_l} = \frac{\pi_a}{1-\pi_a} \frac{-\pi_v \exp\left[\beta\left(v_l - c\right) a_h\right] \lambda_{v_h} + \left(1-\pi_v\right) \exp\left[\beta\left(v_h - c\right) a_h\right] \lambda_{v_l}}{\pi_v \exp\left[\beta\left(v_l - c\right) a_l\right] \lambda_{v_h} - \left(1-\pi_v\right) \exp\left[\beta\left(v_h - c\right) a_l\right] \lambda_{v_l}}$$

next, the number of entering firms equals.

$$N = \left(1+\beta\right)^{-1} \frac{L}{F + \pi_a f} \pi_a \left(\begin{array}{c} \pi_v \frac{\exp[\beta(v_h - c)a_l]}{\pi_a \exp[\beta(v_h-c)a_h] + s_{a_l}(1-\pi_a)\exp[\beta(v_h-c)a_l]} \\ + \left(1-\pi_v\right) \frac{\exp[\beta(v_l-c)a_l]}{\pi_a \exp[\beta(v_l-c)a_h] + s_{a_l}(1-\pi_a)\exp[\beta(v_l-c)a_l]} \end{array} \right)$$

5.2 Trade and Industry Dynamics

I next open markets to trade and analyze whether the distribution of firms shifts towards higher a firms, or whether average sales increase or decrease with trade. Foreign market access is subject to fixed costs and the possibility to export favors more profitable firms. The central new prediction of the model developed in this paper is that the import competition tends to hit domestic exporters more than non-exporters.

For example, consider the car industry in Italy and Germany and assume that luxury car makers are more profitable than the makers of compact car. Since luxury car makers are more profitable, they select into exporting. In Italy, Ferrari gains from the possibility to export to Germany, while exporting this is not profitable for Fiat. In Germany, Porsche exports, while Volkswagen does not. Does that mean that profits for Ferrari and Porsche increase compared to Fiat and Volkswagen? Not necessarily, since the sales of the luxury car makers also suffer much more from the import competition. Sales of Fiat, however, depend very little on the entry of Porsche, since the two firms compete mostly for a different set of customers. It is, therefore, not certain that exposure to trade induces the composition of firms to shift towards the ex ante more profitable entities.

The argument of this paper has to be seen in relation to the productivity reshuffling argument of Melitz (2003). A widespread misjudgment is the notion that the pro-competitive effect of trade is a result of import competition. While this is true in partial equilibrium, in general equilibrium, the effects described by Melitz (2003) have nothing to do with import competition, but are the exclusive result of the possibility to export. Import competition does not affect the composition of firms due to the assumed CES preference structure, which results in losses from import competition being proportional to sales. In general equilibrium, the number of entrants is such that firms on average break even. If import competition decreases profits by a certain amount, the number of potential entrants must exactly offset this loss for the free entry condition to still hold true. But, the number of domestic entrants also affects all domestic firms equally. With CES preferences, import competition therefore has no effect on industry competition.

Denote all equilibrium variables in the open economy by a $*$ superscript, i.e., the number of entrants by N^* and revenues in the open economy by $\Pi^*(a_h)$ and $\Pi^*(a_h)$. Last, denote the inverse price indices under trade by A_l^* and A_h^* respectively. I will again assume that also in the open economy, a_h are more profitable. This is a somewhat more binding assumption than the one made in the previous section, since it implies that even with foreign export condition – under the assumption of symmetrical countries also concentrated in a_h firms – a_h are more profitable.

With these assumptions, h firms sell profits in their home market and in n-1 export markets. In contrast, l firms only sell at home. Total sales equal

$$\begin{aligned}\Pi^*(a_l) &= L\left(\pi_v \frac{\exp[\beta(v_h - c)a_l]}{A_l^*} + (1-\pi_v)\frac{\exp[\beta(v_l - c)a_l]}{A_h^*}\right) \\ \Pi^*(a_h) &= \left(1 + (n-1)\tau^{-\beta}\right) L\left(\pi_v \frac{\exp[\beta(v_h - c)a_h]}{A_l^*} + (1-\pi_v)\frac{\exp[\beta(v_l - c)a_h]}{A_h^*}\right)\end{aligned}$$

where competition is now tougher due to the presence of foreign firms. In the presence of n symmetric countries and under the assumption that only h firms export, the price indices become

$$\begin{aligned}\frac{A_l^*}{N^*} &= \pi_a\left(1 + \tau^{-\beta}(n-1)\right)\exp[\beta(v_h - c)a_h] + s_{a_l}^*(1-\pi_a)\exp[\beta(v_h - c)a_l] \\ \frac{A_h^*}{N^*} &= \pi_a\left(1 + \tau^{-\beta}(n-1)\right)\exp[\beta(v_l - c)a_h] + s_{a_l}^*(1-\pi_a)\exp[\beta(v_l - c)a_l]\end{aligned}$$

Free entry determines the number of potential entrants. Again, since only h type firms are profitable, free entry is determined by profits conditional on being a h type firm times the probability of being of h type.

$$\pi_a\left[(1+\beta)^{-1}\Pi^*(a_h) - (f + (n-1)t)\right] = F$$

And entry in the open economy equals

$$N^* = \frac{\left(1 + (n-1)\tau^{-\beta}\right)}{\frac{F}{\pi_a} + f + t(n-1)} L\left(\pi_v \frac{\exp[\beta(v_h - c)a_h]}{A_l^*} + (1-\pi_v)\frac{\exp[\beta(v_l - c)a_h]}{A_h^*}\right)$$

next, we need to solve for the fraction of surviving l firms from the condition that l firms make zero profit, i.e., that $\Pi^*(a_l) = f$

$$s_{a_l}^* = -\left(1 + \tau^{-\beta}(n-1)\right)\frac{\pi_a}{1-\pi_a}\frac{\pi_v \exp[\beta(v_l - c)a_h]\lambda_h^* + (1-\pi_v)\exp[\beta(v_h - c)a_h]\lambda_l^*}{\pi_v \exp[\beta(v_l - c)a_l]\lambda_h^* + (1-\pi_v)\exp[\beta(v_h - c)a_l]\lambda_l^*}$$

where

$$\lambda_h^* \equiv \left(\exp\left[\beta\left(v_h - c\right)a_h\right] - \exp\left[\beta\left(v_h - c\right)a_l\right]\frac{\frac{F}{\pi_a} + f + \left(n-1\right)t}{f\left(1 + \left(n-1\right)\tau^{-\beta}\right)}\right)$$
$$\lambda_l^* \equiv \left(\exp\left[\beta\left(v_l - c\right)a_h\right] - \frac{\frac{F}{\pi_a} + f + \left(n-1\right)t}{f\left(1 + \left(n-1\right)\tau^{-\beta}\right)}\exp\left[\beta v_l a_l\right]\right)$$

Is $s_{a_l}^*$ necessarily lower than s_{a_l}, i.e., does the distributions of firms shift towards more profitable entities? An easy example will highlight why this is not always the case. If accessing the export market is sufficiently costly, $t\tau^\beta \approx \frac{F}{\pi_a} + f$, so that exporting firms only marginally profit from selecting into the export sector Can this be the case? I do not think so, because the condition that firms select into exporting is

$$\left(1+\beta\right)^{-1} L\left(\pi_v \frac{\exp\left[\beta\left(v_h - c\right)a_h\right]}{A_l^*} + \left(1-\pi_v\right)\frac{\exp\left[\beta\left(v_l - c\right)a_h\right]}{A_h^*}\right) \geq t\tau^\beta$$

While entry is

$$\left(1+\beta\right)^{-1} L\left(\pi_v \frac{\exp\left[\beta\left(v_h - c\right)a_h\right]}{A_l^*} + \left(1-\pi_v\right)\frac{\exp\left[\beta\left(v_l - c\right)a_h\right]}{A_h^*}\right) = \frac{\left(\frac{F}{\pi_a} + f + \left(n-1\right)t\right)}{\left(1 + \left(n-1\right)\tau^{-\beta}\right)}$$

so that we require

$$\frac{F}{\pi_a} + f \geq t\tau^\beta$$

i.e., at the margin, where exporting does happen but firms merely break even, the share of l firms increases when markets are opened to trade.[6]

Because firms pay also a fix costs when they export, trade increase average fixed costs per firm. In order to make up for this, entry into the industry has to adjust.

Proposition 1 *Assume that* $\left(1 + \tau^{-\beta}\left(n-1\right)\right)\exp\left[\beta\left(v_h - c\right)a_h\right] - \exp\left[\beta\left(v_h - c\right)a_l\right]\frac{\frac{F}{\pi_a}+f+t(n-1)}{f} > 0$ *and* $\frac{\frac{F}{\pi_a}+f+t(n-1)}{f\left(1+\tau^{-\beta}(n-1)\right)} > \frac{\frac{F}{\pi_a}+f}{f}$*, Then, opening markets to trade leads to a reduction of average sales per existing firm and to a lower average a.*

Proof. *With trade, the ratio of revenue of h and l type firms is* $\frac{\Pi^*(a_h)}{\Pi^*(a_l)} = \frac{\frac{F}{\pi_a}+f+t(n-1)}{f}$*. Solving for* $s_{a_l}^*$ *yields Which is larger than* s_{a_l} *whenever* $\frac{\frac{F}{\pi_a}+f+t(n-1)}{f\left(1+\tau^{-\beta}(n-1)\right)} > \frac{\frac{F}{\pi_a}+f}{f}$ ■

What is the intuition for why trade can lead to more entry of firms that do not export? Entry depends on the profitability of h type firms only. If their profits decrease more with trade than

[6] It should be noted that trade is possible at even higher levels of transportation costs. Then, only a fraction of h firm exports.

do the profits of non-exporters conditional on a given s_{a_l} , total entry N* decrease substantially, hence leading to higher profits for l type firms.

A very clear cut case is the situation in which $t \rightarrow \Pi^{F*}(a_h)$, i.e. in which h types export, but just break even on exporting. Then, the net profit from exporting is 0, but the additional export competition of foreign h firms hits domestic h firms much more than domestic l firms.

Trade and Welfare. The ultimate question of interest is whether and to what extent trade creates welfare. Since all consumers spend all their money on one good, their expected utility in the closed economy is equal to

$$\begin{aligned}
E[U_i] &= E\left[\max_{j\epsilon N}\frac{w}{p_{i,j}}e^{a_j v_i + x_{i,j}}\right] \\
&= wE\left[\max_{j\epsilon N} e^{a_j(v_i-c)+x_{i,j}}\right] \\
&= w\Gamma(1-\beta)\sum_{j\epsilon N} e^{a_j(v_i-c)\beta} \\
&= N\left(\pi_a\left(1+\tau^{-\beta}(n-1)\right)\exp[\beta(v_i-c)a_h] + s_{a_l}(1-\pi_a)\exp[\beta(v_i-c)a_l]\right) \\
&= NA_i
\end{aligned}$$

(recall the normalization: $w\Gamma(1-\beta)=1$). In the open economy, trade is equal to

$$\begin{aligned}
E[U_i^*] &= wE\left[\max_{j\epsilon N,N_h^*}\frac{1}{p_{i,j}}e^{a_j v_i + x_{i,j}}\right] \\
&= w\Gamma(1-\beta)\left(\sum_{j\epsilon N} e^{a_j(v_i-c)\beta} + \sum_{j\epsilon N_h^*}\tau^{-\beta}e^{a_j(v_i-c)\beta}\right) \\
&= N^* A_i^*
\end{aligned}$$

So we need to compare entry and price index under trade.

$$\begin{aligned}
E[U_h^*] &= \frac{\left(1+(n-1)\tau^{-\beta}\right)}{\frac{F}{\pi_a}+f+t(n-1)}L\left(\pi_v\exp[\beta(v_h-c)a_h]\left(\frac{A_h^*}{A_l^*}\right)+(1-\pi_v)\exp[\beta(v_l-c)a_h]\right) \\
E[U_l^*] &= \frac{\left(1+(n-1)\tau^{-\beta}\right)}{\frac{F}{\pi_a}+f+t(n-1)}L\left(\pi_v\exp[\beta(v_h-c)a_h]+(1-\pi_v)\exp[\beta(v_l-c)a_h]\left(\frac{A_l^*}{A_h^*}\right)\right)
\end{aligned}$$

where

$$\frac{A_h^*}{A_l^*} = \frac{\pi_a\left(1+\tau^{-\beta}(n-1)\right)\exp[\beta(v_h-c)a_h] + s_{a_l}^*(1-\pi_a)\exp[\beta(v_h-c)a_l]}{\pi_a\left(1+\tau^{-\beta}(n-1)\right)\exp[\beta(v_l-c)a_h] + s_{a_l}^*(1-\pi_a)\exp[\beta(v_l-c)a_l]}$$

6 A Continuos Distribution of Attributes

The previous section has that trade can lead more entry of non-exporters. The model, however, is highly reduced and only featured two different types of firms. What are the general properties of the distribution of firms under which trade does lead to a reshuffling of firms towards more profitable entities?

This section demonstrates that whether this is the case depends on how trade affects the ratio of fix costs to industry turnover. While this might seem a purely technical condition at first sight, underlying is a very strong economic intuition. The number of firms depends on expected profits. Since the revenue of the total economy is fixed, whatever the distribution of firms, ex ante, each firm has expected revenues of total industry revenue divided by the number of firms. Expected profits hence depend on the share of profits of revenue. Firms charge constant mark-ups, but also pay fixed costs. Expected profits hence depend on the ratio of sales over fixed costs in the entire economy.

I start by going back to the general demand condition (7).

$$D\left(a_j, p_j\right) = w\Gamma\left(1-\beta\right) \sum_{v_i \epsilon [v_l, v_h]} \frac{p_j^{-(1+\beta)} \exp\left[\beta v_i a_j\right]}{\sum_{n \epsilon J} p_n^{-\beta} \exp\left[\beta v_i a_n\right]}$$

Because firms face a CES type demand, they sell at a constant markup, and their profits equal a share $(1+\beta)^{-1}$ of revenue minus any fixed costs. Thus,

$$E\left[\Pi\left(a_j\right)\right] = E\left[\max\left[w\Gamma\left(1-\beta\right)\frac{\sum_{i\epsilon I}\exp\left[\beta\left(v_i-c\right)a_j\right]}{\sum_{n\epsilon J}\exp\left[\beta\left(v_i-c\right)a_n\right]} - f, 0\right]\right]$$

Valuation draws are distributed with distribution $g_v(v)$ and the number of consumers at each v is thus equal to $\widetilde{L}g_v(v)$. attributes are distributed with distribution $g_a(a)$. Because there may exist realizations of a for which firms choose to exit the industry, the actual distribution of competitors can be different than $g_a(a)$. Let $\Psi(a)$ denote the indicator function that equals one if the firm chooses to be active in the industry.

If there are N firms that pay the fixed cost F and enter the industry, the distribution of active firms is thus equal to $Ng_a(a)\Psi(a)$. We next denote measures of the thoughness of competition, i.e. the pricing index

$$A(v) \equiv \sum_{n\epsilon J}\exp\left[\beta\left(v-c\right)a_n\right] = N\int_{-\infty}^{\infty} g_a(a)\,\Psi(a)\exp\left[\beta\left(v-c\right)a\right]da$$

Thus, denoting $L = w\Gamma(1-\beta)\widetilde{L}$

$$E[\Pi(a_j)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} L g_v(v)\frac{g_a(a)\Psi(a)\exp[\beta(v-c)a]}{A(v)}dvda - f\int_{-\infty}^{\infty} g_a(a)\Psi(a)\,da$$

Changing the order of integration yields a trivial result: because in expectation, each firm is exactly as like to be of a typical a, it will on average have a market share of $1/N$.

$$\begin{aligned} E[\Pi(a_j)] &= L\int_{-\infty}^{\infty} g_v(v)\frac{\int_{-\infty}^{\infty} g_a(a)\Psi(a)\exp[\beta(v-c)a]\,da}{N\int_{-\infty}^{\infty} g_a(a)\Psi(a)\exp[\beta(v-c)a]\,da}dv - f\int_{-\infty}^{\infty} g_a(a)\Psi(a)\,da \\ &= \Gamma(1-\beta)\frac{L}{N} - f\int_{-\infty}^{\infty} g_a(a)\Psi(a)\,da \end{aligned} \tag{10}$$

In order to solve for entry, we thus need to

Definition 1 *Unique cutoff. Assume that*

$$\frac{\partial\int_{-\infty}^{\infty} g_v(v)\frac{\exp[\beta(v-c)a]}{A(v)}dv}{\partial a} = \beta\sum(v-c)\exp[\beta(v-c)a]\frac{g_v(v)}{A(v)}dv > 0$$

Then there is a unique cutoff strategy for which firms choose to exit the industry and hence

$$\Psi(a) = \begin{cases} 1 \text{ if } a \geq a_{\min} \\ 0 \text{ if } a < a_{\min} \end{cases}$$

Lemma 1 *Assume that there exist a Unique cutoff and that the distribution of qualities is given by*

$$g_a(a) = \begin{cases} \lambda\exp[-\lambda a] \text{ if } a \geq 0 \\ 0 \text{ if } a < 0 \end{cases}$$

And that $\lambda > \beta(v_{\max} - c)$ the highest cutoff condition a unique cutoff exists. Then, the number of firms is given by

$$\begin{aligned} \int_0^{\infty}\Psi(a)\lambda\exp[-\lambda a]\,da &= \exp[-\lambda a_{\min}]\,da \\ A(v) &= N\frac{\lambda}{\lambda-\beta(v-c)}\exp[(-\lambda+\beta(v-c))a_{\min}] \end{aligned}$$

so that

$$\int_{-\infty}^{\infty} g_v(v)\frac{\exp[\beta(v-c)a]}{A(v)}dv = \beta\int_{-\infty}^{\infty}\frac{\lambda-\beta(v-c)}{\lambda}(v-c)\exp[\beta(v-c)a]\frac{g_v(v)}{A(v)}dv > 0$$

Proof. *see appendix* ■

6.1 Exit Cutoff

Parameters of the economy can be such that not all firms find it profitable to produce at all. Because - and in stark contrast to productivity - the current model does not display a natural mapping of firm characteristics into an ordering of which firms are more profitable, which firms exit the industry depends on the distribution of valuations in the population, as well as on the cost differences of producing different varieties.

Proposition 2 *(Exit Closed Economy) Assume* $(1-\pi_h)(v_l-c)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)+\pi_h(v_h-c)\left(1-\frac{\beta(v_h-c)}{\lambda}\right)>0$ *and that* $(1+\beta)^{-1}p(a)^{-\beta}\left((1-\pi_h)\frac{1}{A_l}+\pi_h\frac{1}{A_h}\right)L<f$*. Then, there exists a productivity cutoff* $a_{\min}$ *such that*

$$e^{\lambda a_{\min}}=\max\left[\left((1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)+\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)\right)^{-1}(1+\beta)\left(\frac{L}{N}\right)^{-1}f;1\right]$$

below which firms choose to exit the industry.

Proof. *In the closed economy, the two price indices include only domestic firms that do not choose to exit the industry, and hence the marginal firm with quality equal to* $a_{\min}$ *faces the following condition*

$$(1+\beta)^{-1}\left((1-\pi_h)\frac{p(a_{\min})^{-\beta}\exp[\beta v_l a_{\min}]}{N\int_{a_{\min}}^{\infty}g_a(a)p(a)^{-\beta}\exp[\beta v_l a]\,da}+\pi_h\frac{p(a)^{-\beta}\exp[\beta v_h a_{\min}]}{N\int_{a_{\min}}^{\infty}g_a(a)p(a)^{-\beta}\exp[\beta v_h a]\,da}\right)L=f$$

with the cost function $g_a(a)=\lambda e^{-\lambda a}$ *and* $p(a)=e^{ca}$*, this solves to*

$$(1+\beta)^{-1}\left((1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\frac{e^{\beta(v_l-c)a_{\min}}}{e^{(-\lambda+\beta(v_l-c))a_{\min}}}+\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)\frac{e^{\beta(v_h-c)a_{\min}}}{e^{(-\lambda+\beta(v_h-c))a_{\min}}}\right)\frac{L}{N}=f$$

This solves to $e^{\lambda a_{\min}}=\left(1+\frac{\beta}{\lambda}\left((1-\pi_h)(v_l-c)+\pi_h(v_h-c)\right)\right)^{-1}(1+\beta)\left(\frac{L}{N}\right)^{-1}f$*. Second, taking the derivative of profits (16) with respect to quality show that* $\frac{\partial\Pi(a)}{\partial a}|_{a=a_{\min}}>0$ *whenever* $(1-\pi_h)(v_l-c)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)+\pi_h(v_h-c)\left(1-\frac{\beta(v_h-c)}{\lambda}\right)>0$.

$$\frac{\partial\Pi(a)}{\partial a}=(1+\beta)^{-1}\begin{pmatrix}(1-\pi_h)(v_l-c)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)e^{\beta(v_l-c)(a-a_{\min})}\\+\pi_h(v_h-c)\left(1-\frac{\beta(v_h-c)}{\lambda}\right)e^{\beta(v_h-c)(a-a_{\min})}\end{pmatrix}e^{-\lambda a_{\min}}\frac{L}{N}\qquad(11)$$

We note that the first term can be negative, and this is most important at low values of $a=a_{\min}$*. (remember an earlier assumption is that* $\lambda>\beta(v_h-c)$*). It is important to note that while this paper focuses on companies with high a being more productive in autarky, this is not a crucial* *assumption. Next, we need to show that also for the firm with quality $a = 0$, it does not enter, yielding:*

$$(1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)e^{-(-\lambda+\beta(v_l-c))a_{\min}}+\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)e^{-(-\lambda+\beta(vh-c))a_{\min}}<\frac{N}{L}(1+\beta)f$$

where $a_{\min}$ and N are uniquely determined by parameters ■

Corollary 1 *Assume that $(1-\pi_h)(v_l-c)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)+\pi_h(v_h-c)\left(1-\frac{\beta(v_h-c)}{\lambda}\right)<0$ Then, there exists a productivity cutoff $a_{\max}$ such that*

$$\begin{aligned}(1+\beta)^{-1}\left(\frac{N}{L}\right)f &= (1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\left(e^{-\beta(v_h-c)a\max}-e^{-\lambda a_{\max}}\right)^{-1}\\ &\quad+\pi_h\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\left(e^{-\beta(v_h-c)a\max}-e^{-\lambda a_{\max}}\right)^{-1}\end{aligned}$$

Above which firms exit the industry.

Proof. *Assume that all firms above some $a_{\max}$ exit the industry. Then, In the closed economy, the two price indices include only domestic firms that do not choose to exit the industry, and hence the marginal firm with quality equal faces the following condition*

$$(1+\beta)^{-1}\left((1-\pi_h)\frac{p(a_{\max})^{-\beta}\exp[\beta v_l a_{\max}]}{N\int_0^{a_{\max}}g_a(a)p(a)^{-\beta}\exp[\beta v_l a]\,da}+\pi_h\frac{p(a)^{-\beta}\exp[\beta v_h a_{\max}]}{N\int_0^{a_{\max}}g_a(a)p(a)^{-\beta}\exp[\beta v_h a]\,da}\right)L=f$$

$$\left(\begin{array}{c}(1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\left(e^{-\beta(v_h-c)a\max}-e^{-\lambda a_{\max}}\right)^{-1}\\+\pi_h\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\left(e^{-\beta(v_h-c)a\max}-e^{-\lambda a_{\max}}\right)\end{array}\right)(1+\beta)^{-1}=\left(\frac{N}{L}\right)f$$

and the condition that $\frac{\partial\Pi(a)}{\partial a}|_{a=a_{\max}}<0$ is equal to

$$(1+\beta)^{-1}\left(\begin{array}{c}(1-\pi_h)\beta(v_l-c)\frac{\lambda-\beta(v_l-c)}{\lambda\left(e^{-\beta(v_h-c)a\max}-e^{-\lambda a_{\max}}\right)}\\+\pi_h\beta(v_h-c)\frac{\lambda-\beta(v_h-c)}{\lambda\left(e^{-\beta(v_h-c)a\max}-e^{-\lambda a_{\max}}\right)}\end{array}\right)\frac{L}{N}=f$$

■

Note that $A_l=\sum_{n\epsilon J}\Gamma_{j,n}p_n^{-\beta}\exp[\beta v_l a_n]=N\int_{a_{\min}}^{\infty}g_a(a)p(a)^{-\beta}\exp[\beta v_l a]\,da$ and a similar condition for A_h. Thus, we can simplify the condition to equal

$$E[\Pi(a)]=(1+\beta)^{-1}\frac{L}{N}-f(1-G_a(a_{\min}))$$

Note that this simple solution is surprising at first, but very straightforward: total revenue in the industry equals L and profits are a share $(1+\beta)^{-1}$ of revenue. In expectation, the market share of an entrant is simply $1/N$ since all firms are ex ante equal. Next, incorporating the fixed cost, note that with the assumed distribution of cost shocks, $\int_{a_{\min}}^{\infty} g_a(a)\, p(a)^{-\beta} \exp[\beta v_l a]\, da = \frac{\lambda}{\lambda-\beta(v_l-c)} e^{(-\lambda+\beta(v_l-c))a_{\min}}$, whereas $(1-G_a(a_{\min}))\, f = e^{-\lambda a_{\min}} f$ hence solving for expected profits

$$
\begin{aligned}
E[\Pi(a)] &= (1+\beta)^{-1}\frac{L}{N} - (1+\beta)^{-1}\frac{L}{N}\left((1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)+\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)\right) \\
&= (1+\beta)^{-1}\frac{L}{N}\left((1-\pi_h)\frac{\beta(v_l-c)}{\lambda}+\pi_h\frac{\beta(v_h-c)}{\lambda}\right)
\end{aligned}
$$

What explains the GE elements:

- Expected firm sales are proportional to $\frac{L}{N}$
- (omitting fixed costs) Profit as a share of revenue is equal to $(1+\beta)^{-1}$
- (fixed costs) Firms make a high profit when their valuation draw a is a lot higher than the cutoff level amin. Thus, the smaller λ, the more spread the distribution of a draws and the higher profits are in expectation next of the fixed cost. Second, the higher c, the relatively more expensive high a goods are, and thus the smaller the difference between the profits the amin firm makes and a a>amin firm make
- Last, high valuation consumers will tend to buy high a goods much more often than low valuation cost draws. explain

We can now solve for GE by applying the free entry condition: firms on average must break even. Denoting the fixed cost of entry by F.

$$
N = (1+\beta)^{-1}\frac{L}{F}\left((1-\pi_h)\frac{\beta(v_l-c)}{\lambda}+\pi_h\frac{\beta(v_h-c)}{\lambda}\right)
$$

so GE entry cutoff is equal to:

$$
e^{\lambda a_{\min}} = \frac{1-(1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)+\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)}{(1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)+\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)}
$$

Note that this is always positive if:

$$
1 > (1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)+\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right) > 0
$$

which is satisfied by our two conditions:

$$\begin{aligned}(1-\pi_h)(v_l-c)+\pi_h(v_h-c) &> 0\\ \frac{\beta(v_h-c)}{\lambda} &< 1\end{aligned}$$

also note that

$$\begin{aligned}(1-\pi_h)(v_l-c)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)+\pi_h(v_h-c)\left(1-\frac{\beta(v_h-c)}{\lambda}\right) &> 0\\ (1-\pi_h)(v_l-c)+\pi_h(v_h-c) &> 0\end{aligned}$$

7 The Open Economy

With the assumptions made so far, a very strong result can be established: trade between symmetric countries does affect the number of potential entrants to the economy. This seems counter-intuitive, at first, but is actually a trivial result: stemming from the fact that an exponential distribution that is cut off below a certain lower bound is again an exponential distribution.

Proposition 3 *Trade between symmetric countries does not affect the number of entrants if the distribution of attributes is exponential (but the number of firms!). Denote the ex ante cost of entry by F. Irrespective of whether markets are open to trade or not, the equilibrium number of entrants is equal to*

$$N=(1+\beta)^{-1}\frac{L}{F}\frac{\beta}{\lambda}\left((1-\pi_h)(v_l-c)+\pi_h(v_h-c)\right)$$

Proof. *Expected profits and the cost of entry equal*

$$E\left(\Pi(a)+\Pi^*(a)\right)=\int_{a_{\min}}^{\infty}g_a(a)\left(\Pi(a)-f\right)da+(n-1)\int_{a^*_{\min}}^{\infty}g_a(a)\left(\Pi^*(a)-t\right)da$$

Part one of $E\left(\Pi(a)+\Pi^*(a)\right)$:

$$\int_{a_{\min}}^{\infty}g_a(a)\left(\Pi(a)-f\right)da=(1+\beta)^{-1}L\int_{a_{\min}}^{\infty}g_a(a)\begin{pmatrix}(1-\pi_h)\frac{\exp[\beta(v_l-c)a]}{A_l}\\ +\pi_h\frac{\exp[\beta(v_h-c)a]}{A_h}-f\end{pmatrix}da$$

Under the assumption that $a^*_{\min}>a_{\min}$, $a_{\min}$ is such that $\Pi(a_{\min})=f$. Then, note that

$$\begin{aligned}\int_{a_{\min}}^{\infty}g_a(a)\Pi(a)\,da &= (1+\beta)^{-1}L\int_{a_{\min}}^{\infty}\lambda e^{-\lambda a}\begin{pmatrix}(1-\pi_h)\frac{\exp[\beta(v_l-c)a]-\exp[\beta(v_l-c)a_{\min}]}{A_l}\\ +\pi_h\frac{\exp[\beta(v_h-c)a]-\exp[\beta(v_h-c)a_{\min}]}{A_h}\end{pmatrix}da\\ &= \frac{(1+\beta)^{-1}L(1-\pi_h)}{A_l}\left(\frac{\lambda}{\lambda-\beta(v_l-c)}-1\right)e^{(\beta(v_l-c)-\lambda)a_{\min}}\\ &\quad+\frac{(1+\beta)^{-1}L\pi_h}{A_h}\left(\frac{\lambda}{\lambda-\beta(v_h-c)}-1\right)e^{(\beta(v_h-c)-\lambda)a_{\min}}\end{aligned}$$

similarly,

$$(n-1)\int_{a^*_{\min}}^{\infty} g_a(a)\left(\Pi^*(a)-t\right)da = \frac{(1+\beta)^{-1}L(1-\pi_h)}{A_l\tau^{\beta}}(n-1)\left(\frac{\lambda}{\lambda-\beta(v_l-c)}-1\right)e^{(\beta(v_l-c)-\lambda)a^*_{\min}} + \frac{(1+\beta)^{-1}L\pi_h}{A_h\tau^{\beta}}(n-1)\left(\frac{\lambda}{\lambda-\beta(v_h-c)}-1\right)e^{(\beta(v_h-c)-\lambda)a^*_{\min}}$$

Next, solving for A_l and A_h note that:

$$\begin{aligned} A_l &= \sum_{n\epsilon J} p_n^{-\beta}\exp\left[\beta v_l a_n\right] = N\int_{a_{\min}}^{\infty} g_a(a)\exp\left[\beta(v_l-c)a\right]da + N(n-1)\int_{a^*_{\min}}^{\infty} g_a(a)\tau^{-\beta}\exp\left[\beta(v_l-c)a\right]da \\ &= N\frac{\lambda}{\lambda-\beta(v_l-c)}\left(e^{(\beta(v_l-c)-\lambda)a_{\min}}+(n-1)\tau^{-\beta}e^{(\beta(v_l-c)-\lambda)a^*_{\min}}\right) \end{aligned}$$

similarly, $A_h = N\frac{\lambda}{\lambda-\beta(v_h-c)}\left(e^{(\beta(v_h-c)-\lambda)a_{\min}}+(n-1)\tau^{-\beta}e^{(\beta(v_h-c)-\lambda)a^*_{\min}}\right)$

$$\begin{aligned} E\left(\Pi(a)+\Pi^*(a)\right) &= (1+\beta)^{-1}\frac{L}{N}(1-\pi_h)\frac{\beta(v_l-c)}{\lambda}\frac{e^{(\beta(v_l-c)-\lambda)a_{\min}}+(n-1)\tau^{-\beta}e^{(\beta(v_l-c)-\lambda)a^*_{\min}}}{e^{(\beta(v_l-c)-\lambda)a_{\min}}+(n-1)\tau^{-\beta}e^{(\beta(v_l-c)-\lambda)a^*_{\min}}} \\ &\quad +(1+\beta)^{-1}\frac{L}{N}\pi_h\frac{\beta(v_h-c)}{\lambda}\frac{\left(e^{(\beta(v_h-c)-\lambda)a^*_{\min}}+(n-1)\tau^{-\beta}e^{(\beta(v_h-c)-\lambda)a_{\min}}\right)}{\left(e^{(\beta(v_h-c)-\lambda)a_{\min}}+(n-1)\tau^{-\beta}e^{(\beta(v_h-c)-\lambda)a^*_{\min}}\right)} \\ &= (1+\beta)^{-1}\frac{L}{N}\frac{\beta}{\lambda}\left((1-\pi_h)(v_l-c)+\pi_h(v_h-c)\right) \end{aligned}$$

The first two terms simplify because profits are a share $(1+\beta)^{-1}$ of sales. On expectation, all firms make the same sales. $\frac{\exp[\beta v_l a]}{\sum_{n\epsilon J} p_n^{-\beta}\exp[\beta v_l a_n]}$ is a measure of the market share of a potential entrant with $a_n = a$. On expectation, it is $1/N$ Since total market size is nL, n does not matter and expected sales equal $E\left(\Pi(a)+\Pi^*(a)\right)$. Next, note that free entry ensures that

$$F = (1+\beta)^{-1}\frac{L}{N}\frac{\beta}{\lambda}\left((1-\pi_h)(v_l-c)+\pi_h(v_h-c)\right)$$

■

(Keep in mind that the condition that high firms are alive & export is $(1-\pi_h)(v_l-c)+\pi_h(v_h-c)>0$ so expected profits are always positive.

Appendix: the case with general $c(a)$ and $g_a(a)$

Lemma: if f and t equal 0, it is always the case that entry is $(1+\beta)^{-1}\frac{L}{N}=F$

Then point out Point out the why the exponential solves this conveniently.

$$(1-G_a(a_{\min}))\,p(a_{\min})^{-\beta}\exp\left[\beta v_l a_{\min}\right] = \exp\left[-\lambda a_{\min}+\beta(v_l-c)a_{\min}\right]$$

whereas

$$\int_{a_{\min}}^{\infty} g_a(a)\, p(a)^{-\beta} \exp[\beta v_l a]\, da = \frac{\lambda}{\lambda - \beta(v_l - c)} \exp[-\lambda a_{\min} + \beta(v_l - c)\, a_{\min}]$$

What is the name of functions that have the property that you cut of the lower tail and they again look like this function (Pareto and exponential)?

7.1 Entry and Export Cutoffs under Trade

Entry (alive at home):

$$(1+\beta)^{-1}\left((1-\pi_h)\frac{p(a_{\min})^{-\beta}\exp[\beta v_l a_{\min}]}{A_l} + \pi_h \frac{p(a_{\min})^{-\beta}\exp[\beta v_h a_{\min}]}{A_h}\right)L = f$$

Exporting firms (foreign):

$$(1+\beta)^{-1}\left((1-\pi_h)\frac{p^*(a^*_{\min})^{-\beta}\exp[\beta v_l a^*_{\min}]}{A_l} + \pi_h \frac{p^*(a^*_{\min})^{-\beta}\exp[\beta v_h a^*_{\min}]}{A_h}\right)L = t$$

Where

$$\begin{aligned} A_l &= N\frac{\lambda}{\lambda-\beta(v_l-c)}\left(e^{(-\lambda+\beta(v_l-c))a_{\min}} + (n-1)\,\tau^{-\beta}e^{(-\lambda+\beta(v_l-c))a^*_{\min}}\right) \\ A_h &= N\frac{\lambda}{\lambda-\beta(v_h-c)}\left(e^{(-\lambda+\beta(v_h-c))a_{\min}} + (n-1)\,\tau^{-\beta}e^{(-\lambda+\beta(v_h-c))a^*_{\min}}\right) \end{aligned}$$

entry (alive at home)

$$(1+\beta)^{-1}\left(\begin{array}{c} (1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\frac{e^{\beta(v_l-c)a_{\min}}}{e^{(-\lambda+\beta(v_l-c))a_{\min}}+(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_l-c))a^*_{\min}}} \\ +\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)\frac{e^{\beta(v_h-c)a_{\min}}}{e^{(-\lambda+\beta(v_h-c))a_{\min}}+(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_h-c))a^*_{\min}}} \end{array}\right)\frac{L}{N} = f$$

Generally, the two cutoffs are determined by:

$$(1+\beta)^{-1}\left(\begin{array}{c} (1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\frac{(n-1)\tau^{-\beta}}{e^{-\beta(v_l-c)\left(a^*_{\min}-a_{\min}\right)}+(n-1)\tau^{-\beta}e^{-\lambda\left(a^*_{\min}-a_{\min}\right)}} \\ +\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)\frac{(n-1)\tau^{-\beta}}{e^{-\beta(v_h-c)\left(a^*_{\min}-a_{\min}\right)}+(n-1)\tau^{-\beta}e^{-\lambda\left(a^*_{\min}-a_{\min}\right)}} \end{array}\right)e^{\lambda a_{\min}}\frac{L}{N} = (n-1)\,t$$

$$(1+\beta)^{-1}\left(\begin{array}{c}(1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\frac{1}{1+(n-1)\tau^{-\beta}e^{\left(-\lambda+\beta(v_l-c)\right)\left(a^*_{\min}-a_{\min}\right)}}\\ +\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)\frac{1}{1+(n-1)\tau^{-\beta}e^{\left(-\lambda+\beta(v_h-c)\right)\left(a^*_{\min}-a_{\min}\right)}}\end{array}\right)e^{\lambda a_{\min}}\frac{L}{N}=f$$

Dividing upper by lower, we get:

$$\frac{(1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\frac{e^{\beta(v_l-c)\left(a^*_{\min}-a_{\min}\right)}}{1+(n-1)\tau^{-\beta}e^{\left(-\lambda+\beta(v_l-c)\right)\left(a^*_{\min}-a_{\min}\right)}}+\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)\frac{e^{\beta(v_h-c)\left(a^*_{\min}-a_{\min}\right)}}{1+(n-1)\tau^{-\beta}e^{\left(-\lambda+\beta(v_h-c)\right)\left(a^*_{\min}-a_{\min}\right)}}}{(1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\frac{1}{1+(n-1)\tau^{-\beta}e^{\left(-\lambda+\beta(v_l-c)\right)\left(a^*_{\min}-a_{\min}\right)}}+\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)\frac{1}{1+(n-1)\tau^{-\beta}e^{\left(-\lambda+\beta(v_h-c)\right)\left(a^*_{\min}-a_{\min}\right)}}}$$

$$=\frac{t\tau^{\beta}}{f}$$

Proposition: If all consumers are homogenous then $a_{\min}$ is increasing in t and decreasing in τ. One special case is the standard productivity heterogeneity model where $v_l = v_h = v > c$. Then,

$$e^{\beta(v-c)\left(a^*_{\min}-a_{\min}\right)}=\frac{t\tau^{\beta}}{f}\quad\text{and}\quad e^{\lambda a_{\min}}=\frac{N}{L}f\left(1-\frac{\beta(v-c)}{\lambda}\right)^{-1}\left(1+(n-1)\,\tau^{-\beta}\left(\frac{t\tau^{\beta}}{f}\right)^{-\frac{\lambda-\beta(v-c)}{\beta(v-c)}}\right)$$

Next lets characterize $a^*_{\min}-a_{\min}$ in the case of heterogeneity. The condition that $a^*_{\min}>a_{\min}$ is that $\frac{t\tau^{\beta}}{f}>1$. Assume this is true. Then, rewriting (12) yields

$$\begin{aligned}v\left(a^*_{\min}-a_{\min},t\right)\equiv\;&(1-\pi_h)\left(1-\frac{\beta\left(v_l-c\right)}{\lambda}\right)\frac{e^{\beta(v_l-c)\left(a^*_{\min}-a_{\min}\right)}-\frac{t\tau^{\beta}}{f}}{1+(n-1)\,\tau^{-\beta}e^{\left(-\lambda+\beta(v_l-c)\right)\left(a^*_{\min}-a_{\min}\right)}}\\&+\pi_h\left(1-\frac{\beta\left(v_h-c\right)}{\lambda}\right)\frac{e^{\beta(v_h-c)\left(a^*_{\min}-a_{\min}\right)}-\frac{t\tau^{\beta}}{f}}{1+(n-1)\,\tau^{-\beta}e^{\left(-\lambda+\beta(v_h-c)\right)\left(a^*_{\min}-a_{\min}\right)}}\end{aligned}$$

It is neither clear that $\frac{\partial v\left(a^*_{\min}-a_{\min},t\right)}{\partial t}<0$ nor that $\frac{\partial v\left(a^*_{\min}-a_{\min},t\right)}{\partial\left(a^*_{\min}-a_{\min}\right)}>0$ when $v_l<c$. Indeed, consider the case of a negative v_l (that of course has to still satisfy the conditions assumed above).

$$\frac{\partial v\left(a^*_{\min}-a_{\min},t\right)}{\partial\left(a^*_{\min}-a_{\min}\right)} =$$

$$= (1-\pi_h)\frac{\lambda-\beta(v_l-c)}{\lambda}$$

$$\left(\begin{array}{c} \beta(v_l-c)\frac{e^{\beta(v_l-c)(a^*_{\min}-a_{\min})}}{1+(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_l-c))\left(a^*_{\min}-a_{\min}\right)}}- \\ (-\lambda+\beta(v_l-c))(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_l-c))\left(a^*_{\min}-a_{\min}\right)}\frac{\left(e^{\beta(v_l-c)(a^*_{\min}-a_{\min})}-\frac{t\tau^\beta}{f}\right)}{\left(1+(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_l-c))\left(a^*_{\min}-a_{\min}\right)}\right)^2} \end{array}\right)$$

$$+\pi_h\frac{\lambda-\beta(v_h-c)}{\lambda}$$

$$\left(\begin{array}{c} \frac{\beta(v_h-c)e^{\beta(v_h-c)(a^*_{\min}-a_{\min})}}{1+(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_h-c))\left(a^*_{\min}-a_{\min}\right)}}- \\ (-\lambda+\beta(v_h-c))(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_h-c))\left(a^*_{\min}-a_{\min}\right)}\frac{\left(e^{\beta(v_h-c)(a^*_{\min}-a_{\min})}-\frac{t\tau^\beta}{f}\right)}{\left(1+(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_h-c))\left(a^*_{\min}-a_{\min}\right)}\right)^2} \end{array}\right)$$

Lets evaluate this at $a^*_{\min}-a_{\min}=0$.

$$\left.\frac{\partial v\left(a^*_{\min}-a_{\min},t\right)}{\partial\left(a^*_{\min}-a_{\min}\right)}\right|_{a^*_{\min}-a_{\min}=0,}\frac{t\tau^\beta}{f}=1 = (1-\pi_h)\frac{\lambda-\beta(v_l-c)}{\lambda}\beta(v_l-c)\frac{1}{1+(n-1)\tau^{-\beta}}+ \pi_h\frac{\lambda-\beta(v_h-c)}{\lambda}\beta(v_h-c)\frac{1}{1+(n-1)\tau^{-\beta}}$$

By assumption, this is positive. (And it is also positive for all $a^*_{\min}-a_{\min}>0$)

$$\left.\frac{\partial v\left(a^*_{\min}-a_{\min},t\right)}{\partial t}\right|_{a^*_{\min}-a_{\min}=0,}\frac{t\tau^\beta}{f}=1$$

$$= (1-\pi_h)\left(1-\frac{\beta(v_l-c)}{\lambda}\right)\frac{-\frac{\tau^\beta}{f}}{1+(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_l-c))\left(a^*_{\min}-a_{\min}\right)}} +\pi_h\left(1-\frac{\beta(v_h-c)}{\lambda}\right)\frac{-\frac{\tau^\beta}{f}}{1+(n-1)\tau^{-\beta}e^{(-\lambda+\beta(v_h-c))\left(a^*_{\min}-a_{\min}\right)}}$$

what if $a^*_{\min}-a_{\min}$ is very large? All $e^{(-\lambda+\beta(v_h-c))\left(a^*_{\min}-a_{\min}\right)}$ go to 0, lets assume $(v_l-c)<0$ and that but it is not clear that $e^{(-\lambda+\beta(v_h-c))\left(a^*_{\min}-a_{\min}\right)}e^{\beta(v_h-c)\left(a^*_{\min}-a_{\min}\right)}$

$$
\begin{aligned}
& \frac{\partial v\left(a_{\min}^{*}-a_{\min},t\right)}{\partial\left(a_{\min}^{*}-a_{\min}\right)}\Bigg|_{a_{\min}^{*}-a_{\min}\to\infty,\ \frac{t\tau^{\beta}}{f}=1} \\
= & \left(1-\pi_h\right)\frac{\lambda-\beta\left(v_l-c\right)}{\lambda}\left(\frac{\beta\left(v_l-c\right)\frac{e^{\beta\left(v_l-c\right)\left(a_{\min}^{*}-a_{\min}\right)}}{1}-\left(-\lambda+\beta\left(v_l-c\right)\right)}{\left(n-1\right)\tau^{-\beta}e^{\left(-\lambda+\beta\left(v_l-c\right)\right)\left(a_{\min}^{*}-a_{\min}\right)}\frac{\left(e^{\beta\left(v_l-c\right)\left(a_{\min}^{*}-a_{\min}\right)}-\frac{t\tau^{\beta}}{f}\right)}{1}}\right) \\
& +\pi_h\frac{\lambda-\beta\left(v_h-c\right)}{\lambda}\left(\frac{\frac{\beta\left(v_h-c\right)e^{\beta\left(v_h-c\right)\left(a_{\min}^{*}-a_{\min}\right)}}{1}-\left(-\lambda+\beta\left(v_h-c\right)\right)}{\left(n-1\right)\tau^{-\beta}e^{\left(-\lambda+\beta\left(v_h-c\right)\right)\left(a_{\min}^{*}-a_{\min}\right)}\frac{\left(e^{\beta\left(v_h-c\right)\left(a_{\min}^{*}-a_{\min}\right)}-\frac{t\tau^{\beta}}{f}\right)}{1}}\right)
\end{aligned}
$$

8 Conclusion

This paper develops a model featuring endogenous market segmentation, demand heterogeneity, and firm exit and entry. I analyze the dynamics of industry when competition increases due to higher import competition. Most importantly, I challenge the view that the distribution of active firms shifts towards more profitable entities when markets are opened to trade.

The entry and exit decision of firms into industry depends on firm profitability rather than on physical productivity differences. Indeed, the empirical literature suggests that demand heterogeneity is a far more important determinant of industry composition than are differences in physical productivity. The theoretical literature has so far disregarded this fact based on the assertion that any type of firm heterogeneity is equivalent to heterogeneity in physical productivity. I document that this is not necessarily the case.

The mechanism of this paper rests on the premise that there is a reason why different firms face different demand: firm's output differs in its attributes (attributes can be seen in a wide sense for example good quality, location of the supplier, but also color) and the economy is populated by consumers that are heterogeneous in their valuation or taste for these attributes. In equilibrium, some firms are more profitable because many consumers highly value the attributes of the good they produce.

I next open markets to trade. Consistent with Melitz (2003), I show that the firms that are profitable in the domestic economy select into exporting. In contrast to the existing literature, however, it is not certain that these exporters also gain from trade while the non-exporters loose. Imagine, for example, an industry with red and blue car producers and assume that blue cars producers are more profitable and consequently export when markets are opened to trade. The possibility to export allows an extra profit for producers of blue cars, but also foreign blue car producers will select into exporting. The domestic market for blue cars will thus get much tougher with trade, while the market for red cars is only mildly affected. It is thus not clear that those producers do not export loose compared to exporters. In general equilibrium, it can even be true that the firms that do not export gain most from globalization.

9 Appendix 1

9.1 Demand and Solution to the Gumbel

$$\begin{aligned}
D_j(a_j,p_j,v_j) &= \int\limits_{v\epsilon V} j_v(v) \int\limits_{x_{i,j}\epsilon X} \frac{I_v}{p_j} j_x(x_{i,j}) \Pr\left(\frac{e^{a_j v + x_{i,j}}}{p_j} = \max_{n\epsilon J}\frac{e^{a_n v + x_{in}}}{p_n}\right) dx_{i,j} dv \\
&= \int\limits_{v\epsilon V} j_v(v) \int\limits_{x_{i,j}\epsilon X} \frac{I_v}{p_j} j_x(x_{i,j}) \Pr\left(\max_{n\epsilon J, n\neq j}\frac{e^{a_n v + x_{in}}}{p_n} < \frac{e^{a_j v + x_{i,j}}}{p_j}\right) dx_{i,j} dv \\
&= \int\limits_{v\epsilon V} j_v(v) \int\limits_{x_{i,j}\epsilon X} \frac{I_v}{p_j} j_x(x_{i,j}) \prod_{n\neq j} \Pr\left(\frac{e^{a_n v + x_{in}}}{p_n} < \frac{e^{a_j v + x_{i,j}}}{p_j}\right) dx_{i,j} dv \\
&= \int\limits_{v\epsilon V} j_v(v) \int\limits_{x_{i,j}\epsilon X} \frac{I_v}{p_j} j_x(x_{i,j}) \prod_{n\neq j} \Pr\left(x_{i,n} < \ln(p_n) - \ln(p_j) + (a_j - a_n) v + x_{i,j}\right) dx_{i,j} dv
\end{aligned}$$

So, if all x are distributed Gumbel with scale parameter 0 and shape parameter $1/\beta$. (the scale parameter can be altered in any way, this does not change anything)

$$\begin{aligned}
j_x(x) &= \exp\left[-\exp\left[-x\beta\right]\right] \\
j_x(x) &= \frac{1}{\beta}\exp\left[-x\beta\right]\exp\left[-\exp\left[-x\beta\right]\right]
\end{aligned}$$

The math works out to be:

$$\begin{aligned}
\Pr\left(x_{i,n} < \ln(p_n) - \ln(p_j) + (a_j - a_n) v + x_{i,j}\right) &= \exp\left[-\exp\left[-\beta\left(\ln(p_n) - \ln(p_j) + (a_j - a_n) v + x_{i,j}\right)\right]\right] \\
&= \exp\left[-p_j^{\beta} p_n^{-\beta} \exp\left[-\beta\left(a_j v + x_{i,j}\right)\right] \exp\left[v\beta q_n\right]\right]
\end{aligned}$$

so that

$$= \prod_{n\neq j} \Pr\left(x_{i,n} < \ln(p_n) - \ln(p_j) + (a_j - a_n) v + x_{i,j}\right) = \exp\left[-p_j^{1/\beta} \exp\left[-\beta\left(a_j v + x_{i,j}\right)\right] \sum_{J\neq j}\left(p_n^{-\beta}\exp\left[v\beta q_n\right]\right)\right]$$

so to get demand for each v

$$\begin{aligned}
&= \int\limits_{x_{i,j}\epsilon X} \frac{I_v}{p_j} j_x(x_{i,j}) \prod_{n\neq j} \Pr\left(x_{i,n} < \ln(p_n) - \ln(p_j) + (a_j - a_n) v + x_{i,j}\right) dx_{i,j} \\
&= \frac{I_v}{p_j}\frac{1}{\beta} \int\limits_{x_{i,j}\epsilon X} \exp\left[-\beta x_{i,j}\right] \exp\left[-\exp\left[-\exp -\beta x_{i,j}\right]\left(1 + p_j^{\beta}\exp\left[-\beta v q_j\right]\sum_{J\neq j}\left(p_n^{-\beta}\exp\left[v\beta q_n\right]\right)\right)\right] dx_{i,j}
\end{aligned}$$

Note that : $\left(1+p_j^{\beta}\exp\left[-v\beta q_j\right]\sum_{J\neq j}\left(p_n^{-\beta}\exp\left[v\beta q_n\right]\right)\right)=\sum_{J\neq j}\left(p_n^{-\beta}\exp\left[v\beta q_n\right]\right)$

now, we have to substitute: $z_{i,j}=p_j^{\beta}\exp\left[-v\beta q_j\right]\sum_{j\epsilon J}\left(p_n^{-\beta}\exp\left[v\beta q_n\right]\right)x_{i,j}$

$$=\frac{I_v}{p_j}\frac{1}{\beta}\left(p_j^{\beta}\exp\left[-v\beta q_j\right]\sum_{j\epsilon J}\left(p_n^{-\beta}\exp\left[v\beta q_n\right]\right)\right)^{-1}\int\limits_{z_{i,j}\epsilon X}\exp\left[-z_{i,j}\beta\right]\exp\left[-\exp\left[-\beta z_{i,j}\right]\right]dzx_{i,j}$$

so demand is equal to

$$D_j\left(a_j,p_j,v_j\right)=\Gamma\left(1-\beta\right)\frac{I}{p_j}\int\limits_{v\epsilon V}j_v\left(v\right)\frac{p_j^{-\beta}\exp\left[-v\beta q_j\right]}{\sum\limits_{n\epsilon J}\left(p_n^{-\beta}\exp\left[v\beta q_n\right]\right)}dv \tag{14}$$

=> here the properties,

Proposition 4. The density of attribute draws is equal to $G_a\left(a\right)=1-e^{-\lambda a}$ Where λ is sufficiently large ($\lambda>\beta\left(v_h-c\right)$)

$$D\left(a,p\left(a\right)\right)=\Gamma\left(1-\beta\right)\widetilde{L}wp\left(a\right)^{-(1+\beta)}\left(\left(1-\pi_h\right)\frac{\exp\left[\beta v_l a\right]}{\sum\limits_{n\epsilon J}\Gamma_{j,n}p_n^{-\beta}\exp\left[\beta v_l a_n\right]}+\pi_h\frac{\exp\left[\beta v_h a\right]}{\sum\limits_{n\epsilon J}\Gamma_{j,n}p_n^{-\beta}\exp\left[\beta v_h a_n\right]}\right)$$

Where $\Gamma_{j,n}$ is the indicator function that equals one if the firm is active in a market or not. Set $\Gamma\left(1-\beta\right)\widetilde{L}w\equiv L$ and define:

$$A_l\equiv\sum_{n\epsilon J}\Gamma_{j,n}p_n^{-\beta}\exp\left[\beta v_l a_n\right]\quad\text{and}\quad A_h\equiv\sum_{n\epsilon J}\Gamma_{j,n}p_n^{-\beta}\exp\left[\beta v_h a_n\right]$$

A_l and A_h are the equivalent of the price index in CES demand functions adjusted by characteristics and valuation. Profits in the domestic markets are equal to sales times markup, or $D\left(a,p\left(a\right)\right)\left(p\left(a\right)-c\left(a\right)\right)$. Given the CES demand, firms will find it optimal to charge a markup of $p\left(a\right)=\left(1+\beta\right)/\beta c\left(a\right)$. Denote the domestic profit of a firm that charges the optimal price by $\Pi\left(a\right)$. The latter flow equals

$$\Pi\left(a\right)=\left(1+\beta\right)^{-1}p\left(a\right)^{-\beta}\left(\left(1-\pi_h\right)\frac{\exp\left[\beta v_l a\right]}{A_l}+\pi_h\frac{\exp\left[\beta v_h a\right]}{A_h}\right)L-f \tag{15}$$

Denoting the optimized profit flow on all export markets by $\Pi^*\left(a\right)$, which equals

$$\Pi^*\left(a\right)=\left(n-1\right)\left(1+\beta\right)^{-1}p^*\left(a\right)^{-\beta}\left(\left(1-\pi_h\right)\frac{\exp\left[\beta v_l a\right]}{A_l^*}+\pi_h\frac{\exp\left[\beta v_h a\right]}{A_h^*}\right)L-\left(n-1\right)t \tag{16}$$

Where (16) assumes that a firm that is exporting to one market also serves all other export markets. More importantly, due to the assumption of having symmetric countries $A_l^* = A_l$ and $A_h^* = A_h$.

10 Append 2: Heterogeneity in Attributes and Productivity

We next analyze the case with both heterogeneity of products and firm productivity. The marginal cost of selling one good at home and abroad, respectively, equals

$$c_j = \rho_j^{-1} e^{ca_j} \quad \text{and} \quad c_j^* = \rho_j^{-1} \tau e^{ca_j} \tag{17}$$

Where ρ_j is the firm's productivity draw that is distributed Pareto and independent of a_j.

$$F_\rho(\rho) = 1 - \left(\frac{\rho}{\rho_{\min}}\right)^{-\gamma}$$

The Timing of this simple economy is the following:

1. N entrants pay the fixed cost F and enter the industry.

2. Each entrant receives a ρ and an $a-$draw. With probability π_a, the draw is equal to a_h and with probability $1 - \pi_a$ it is equal to a_l. ρ is distributed Pareto $(\rho_{\min}, \gamma)$.

3. Firms below a certain cutoff level $\overline{\rho_i}$ leave the industry. For clearer exposition, we will assume that parameters are such that $\overline{\rho_h} < \overline{\rho_l}$ i.e. that h firms are, on average, more profitable than l firms.

4. All remaining h and l firms produce and pay a fixed cost f to keep their business alive.

5. Firms Produce and Sell.

10.1 Domestic Equilibrium

Each firm faces a demand of

$$D(a_j) = p(a_j)^{-(1+\beta)} L \left(\pi_v \frac{\exp[\beta v_h a_j]}{NA_h} + (1 - \pi_v) \frac{\exp[\beta v_l a_j]}{NA_l}\right)$$

Where A_l^{-1}, A_h^{-1} are the respective price indices for consumers v_l and v_h – the central measure of the thoughness of competition in this economy.

$$
\begin{aligned}
A_h &\equiv \sum_{j\epsilon N} p\left(a_j\right)^{-\beta} \exp\left[\beta v_i a_h\right] \\
A_l &\equiv \sum_{j\epsilon N} p\left(a_j\right)^{-\beta} \exp\left[\beta v_i a_l\right]
\end{aligned}
$$

which is equal to:

$$
\begin{aligned}
A_h &= \sum_{j\epsilon N} p\left(a_j\right)^{-\beta} \exp\left[\beta v_i a_h\right] \\
&= N\pi_a \exp\left[\beta\left(v_h - c\right) a_h\right] \int_{\overline{\rho_h}}^{\infty} \rho^{\beta}\gamma\left(\rho\right)^{-\gamma-1} \rho_{\min}^{\gamma} d\rho + N\left(1-\pi_a\right) \exp\left[\beta\left(v_h - c\right) a_l\right] \int_{\overline{\rho_l}}^{\infty} \rho^{\beta}\gamma\left(\rho\right)^{-\gamma-1} \rho_{\min}^{\gamma} d \\
&= N\frac{\gamma}{\gamma-\beta}\rho_{\min}^{\gamma}\left(\overline{\rho_l}\right)^{-(\gamma-\beta)}\left(\pi_a \exp\left[\beta\left(v_h-c\right)a_h\right]\left(\frac{\overline{\rho_h}}{\overline{\rho_l}}\right)^{-(\gamma-\beta)} + \left(1-\pi_a\right)\exp\left[\beta\left(v_h-c\right)a_l\right]\right)
\end{aligned}
$$

similarly,

$$
A_l = N\frac{\gamma}{\gamma-\beta}\rho_{\min}^{\gamma}\left(\overline{\rho_l}\right)^{-(\gamma-\beta)}\left(\pi_a \exp\left[\beta\left(v_l-c\right)a_h\right]\left(\frac{\overline{\rho_h}}{\overline{\rho_l}}\right)^{-(\gamma-\beta)} + \left(1-\pi_a\right)\exp\left[\beta\left(v_l-c\right)a_l\right]\right)
$$

Lets first solve for $\left(\frac{\overline{\rho_h}}{\overline{\rho_l}}\right)^{-(\gamma-\beta)}$.

Revenue is equal to

$$
\begin{aligned}
\Pi\left(a_h\right) &= \overline{\rho_h}^{-\beta}\frac{L}{N}\left(\pi_v\frac{\exp\left[\beta\left(v_h-c\right)a_h\right]}{A_h} + \left(1-\pi_v\right)\frac{\exp\left[\beta\left(v_l-c\right)a_h\right]}{A_l}\right) = \left(1-\beta\right)f \\
\Pi\left(a_l\right) &= \overline{\rho_l}^{-\beta}\frac{L}{N}\left(\pi_v\frac{\exp\left[\beta\left(v_h-c\right)a_l\right]}{A_h} + \left(1-\pi_v\right)\frac{\exp\left[\beta\left(v_l-c\right)a_l\right]}{A_l}\right) = \left(1-\beta\right)f
\end{aligned}
$$

so that

$$
\left(\frac{\overline{\rho_h}}{\overline{\rho_l}}\right)^{-\beta}\frac{\pi_v\frac{\exp\left[\beta\left(v_h-c\right)a_h\right]}{A_h} + \left(1-\pi_v\right)\frac{\exp\left[\beta\left(v_l-c\right)a_h\right]}{A_l}}{\pi_v\frac{\exp\left[\beta\left(v_h-c\right)a_l\right]}{A_h} + \left(1-\pi_v\right)\frac{\exp\left[\beta\left(v_l-c\right)a_l\right]}{A_l}} = 1
$$

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