Signaling in Auctions among Competitors

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Abstract

We consider a model of oligopolistic firms that have private information about their cost structure. Prior to competing in the market a competitive advantage, i.e., a cost reducing technology, is allocated to a subset of the firms by means of a multi-object auction. After the auction either all bids or only the prices to be paid are revealed to all firms. This provides an opportunity for signaling. Whether there exists an equilibrium in which bids perfectly identify the bidders' costs generally depends on the type and fierceness of the market competition, the specific auction format, and the bid announcement policy.

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1 Introduction

The analysis of a selling mechanism such as an auction is often reduced to a one-stage game where buyers do not meet again in the future. There are, however, a lot of situations where the outcome of an auction crucially affects further interactions among buyers. For instance, buyers might be firms that bid in an auction in order to gain access to a new market or the right to use a new technology that gives them a competitive advantage. If the firms taking part in the auction are at the same time also rivals in the market for their products, the behavior in the auction is certainly influenced by the expected outcome of future market interactions and vice versa. The auction might not only have an impact on later stages because it changes the market environment by allocating competitive advantages, but also because it might change the informational structure. When firms have private information about demand or cost parameters, participating in the auction can to some extent reveal this information to rivals. In particular, firms might use their bids as signals.

In this paper we analyze a two-stage model of an oligopoly where firms have private information about their costs of production. In the first stage firms bid in a multi-object auction to win access to a cost reducing technology that is limited to a subset of the firms. In the second stage firms then compete in the market. We consider three types of sealed-bid auction rules: the all-pay auction where all bidders are asked to pay their bid, the discriminatory auction where only the winners pay their bid, and the uniform-price auction where the winners all pay the highest losing bid. Do firms in this situation actually use the auction in the first stage as a signaling device to such an extent that bids perfectly identify costs? In order to answer this question we will explore under what circumstances this game has a fully separating equilibrium.

There are several possible applications for our model. As an example for the discriminatory and uniform-price auction, consider an outside innovator who employs one of the two auction rules to sell to firms a limited number of licenses for using a cost reducing innovation.[1] Regarding the all-pay format, we can, e.g., interpret bidding in such an auction as lobbying activities by firms that try to convince politicians to grant them (rather than their competitors) subsidies.[2] Especially in situations where firms disclose their expenses in a lobbying register, using those bids as signals is possible. Similarly, a

[1] Although often used in practice, selling licenses through an auction similar to the ones we consider in this paper is in most cases not the optimal mechanism for the innovator. For the case of a Cournot oligopoly with complete information, Giebe and Wolfstetter (2008) find that the innovator's revenue is maximized by a combination of a license auction with royalty contracts (for both losers and winners).

[2] Lobbying is generally thought to be well represented by an all-pay auction. See, e.g., Baye, Kovenock, and de Vries (1993).

research and development race among firms can be modeled as an all-pay auction.

An important factor affecting the existence of a separating equilibrium is the type of competition in the second stage. In a setting where firms, using a linear technology, produce differentiated products sold to a market with linear demand, we consider both the Cournot model where firms choose quantities as well as the Bertrand model where firms set prices. In both cases the value of winning the auction is higher for low-cost firms. The signaling incentive, however, differs. Under Cournot competition firms aim at understating their costs in order to appear stronger and gain a larger market share. The opposite is true under Bertrand competition. Here, firms prefer to overstate their costs in order for their rivals to set higher prices. In both cases this signaling incentive is strongest for low-cost firms. Hence, under Bertrand competition, firms who would bid highest in the absence of signaling, have also the strongest incentive to reduce their bid for signaling purposes. Therefore, the existence of a separating equilibrium is in general more problematic under Bertrand competition than under Cournot competition where the two effects point into the same direction.

The differences in terms of signaling incentives between the two types of competition have been well known. For example, Gal-Or (1986) studies a model where firms with privately known costs choose the amount of information to be revealed before entering the market competition stage. Gal-Or (1986) finds that in the Bertrand case firms choose to reveal no information at all, whereas in the Cournot model they fully reveal their marginal costs. Ziv (1993) studies pure (costly) signaling in a Cournot market with privately known costs. There, rather than bidding for an object with intrinsic value, firms simply burn money in order to signal their strength which is observed by all competitors. Under Bertrand competition such a separating equilibrium is impossible.

Of course, the existence of a separating equilibrium also depends on how strong signaling incentives are. This depends, in turn, on how much information firms can infer from the auction. The auctioneer might disclose all or only some of the firms' bids. In this paper we concentrate on the cases where either all bids are revealed, or where the amount each firm has to pay is announced. For the all-pay auction there is, of course, no difference among those two possibilities. For the discriminatory auction, announcing the prices to be paid means that only the winners' bids are disclosed, while the highest losing bid alone is revealed in the uniform-price auction. In general, we find that if the auction reveals less information, there are more situations where a separating equilibrium is possible under Bertrand competition.

For the existence of a separating equilibrium it is also important that the payment rules allow for credible signals, i.e., the firms that actually send a signal must also pay accordingly. This is the case in the all-pay auction with all bids revealed and the discriminatory auction with only the winning bids revealed. In those cases the separating equilibrium is likely to exist. When all bids are announced in a discriminatory auction, however, bids from firms with high costs that are pretty sure that they do not have to pay anything are for this reason not very credible. Consequently, a separating equilibrium exists under those circumstances only as a special case.

Closely related to our model is Das Varma (2003) where a cost reducing innovation is allocated among oligopolists through a first-price auction. The amount by which costs are reduced varies among firms and is private information, resulting in an incentive to signal. Das Varma (2003) finds that in the case of Cournot competition there is a unique equilibrium where bids fully reveal all private information, whereas in the Bertrand case such an equilibrium may fail to exist. In a related model, Goeree (2003) extends the analysis of the Cournot case to second-price and English auctions.

In contrast to those authors we assume in this paper that the cost reduction is common knowledge and identical for all firms. Instead, it is ex ante costs that are private information. An important consequence of this is that not only the private information of winners, but also that of losers is relevant for the second stage. An auction of a cost-reducing innovation to Bertrand competitors with private ex ante costs is also studied by Moldovanu and Sela (2003). Yet by assuming that costs always become common knowledge after the auction, they exclude any signaling effects from their model.[3] Katzman and Rhodes-Kropf (2008) consider an auction among firms with private costs as well. In their model, what is allocated through the auction is access to a duopoly with an incumbent firm. Hence, unlike in our model, signals are sent to an outsider rather than to the other bidders. Note that we keep our analysis of the auction stage in Section 3 at a fairly general level, so that, as we show in Subsection 3.5, it also covers several of the models discussed above.

The literature has, so far, almost exclusively focused on single-object auctions. One of the rare exceptions is Katsenos (2007) who compares simultaneous and sequential auctions for selling two licenses that grant access to a duopoly to firms with private costs. In this paper, we analyze multi-object auctions while allowing for the number of winners to be any number smaller than the number of firms. Not only seems the case of multiple winners relevant for applications, such as an innovator selling more than one license, but we also find that the existence of a separating equilibrium on some occasions

[3]Similarly, signaling effects are also excluded from the model of Jehiel and Moldovanu (2000) who, as an example for a more general case, analyze an auction of a privately known cost reduction to Cournot competitors.

crucially depends on the number of winners.

The paper is organized as follows. In Section 2 we present the main assumptions of the model. In Section 3 we develop a general framework for analyzing the bidding behavior in the first stage without having the second stage modeled explicitly, yet. Our specific model of the second stage is then presented in Section 4. In Section 5 we analyze the existence of a separating equilibrium when the auctioneer reveals all bids whereas in Section 6 we consider the case where the prices paid are disclosed. We gather conclusions in Section 7, followed by an appendix containing proofs.

2 The Model

There are n firms that compete in a product market. Firms are all identical except for a firm specific cost parameter c_i. We assume that the lower c_i, the lower are firm i's variable costs. For example, if firms have linear technologies, c_i are the constant marginal costs of firm i. The cost parameter c_i is private information of firm i. It is common knowledge that $c_1, c_2, \ldots, c_n$ are realizations of the random variables $C_1, C_2, \ldots, C_n$ which are independently and identically distributed according to F on the interval $[\underline{c}, \overline{c}]$. The distribution F is twice continuously differentiable, having a strictly positive density $f := F'$. We assume $F''(c) \in \mathbb{R}$ for all $c \in [\underline{c}, \overline{c}]$. In addition, there exists some new technology the use of which generates a competitive advantage. Yet this technology can only be used by $k < n$ firms, access to it being sold through a sealed-bid auction.

The timing of the game is as follows. In the first stage firms submit their bids and the auctioneer determines the k winners of the auction. In addition to the identities of the winners, the auctioneer also publicly reveals the values of a subset of the bids according to a commonly known announcement rule. Then, all firms enter into the second stage of the game where they compete in the product market. When choosing their action in the second stage, firms update their beliefs about their competitors' cost parameters according to what they learn from the auctioneer's announcement in the first stage.

We focus on *separating* equilibria, i.e., on equilibria where a firm's bidding strategy prescribes a different amount for each type. As equilibrium concept we adopt the *symmetric perfect Bayesian Nash equilibrium*, where symmetric means that ex ante all firms use the same equilibrium bidding strategy. In order to find an equilibrium of the whole game, we typically solve for the Bayesian Nash equilibrium of the market interaction in the second stage given the beliefs firms might hold after having played the first stage. From this we obtain the expected payoffs of firms when they choose their bid in the first stage.

Whether or not a firm wins the auction and gains access to the new technology typically depends on the ranking of the firms in terms of their cost parameters. Also, whose bids the auctioneer reveals depends on that ranking. Hence, in the course of its decision-making, firm i has to form expectations about the ranking (and values) of the cost parameters of its rivals. The following definitions will therefore be of great use throughout the paper. Define $\mathcal{C}^{-i} := \{C_1, C_2, \ldots, C_n\} \setminus C_i$ to be the set of cost parameters of the competitors of firm i. Let $Z_1^i, Z_2^i, \ldots, Z_{n-1}^i$ be a rearrangement of all $C_j \in \mathcal{C}^{-i}$ so that $Z_1^i \leq Z_2^i \leq \cdots \leq Z_{n-1}^i$. Consequently, $\mathbf{Z}^i := \left(Z_1^i, Z_2^i, \ldots, Z_{n-1}^i\right)$ is the vector of order statistics of the cost parameters of firm i's rivals. Note that because $C_1, C_2, \ldots, C_n$ are independently and identically distributed, we can drop superscript i in the following statements. The joint density of the order statistics $\mathbf{Z}$ is

$$g_{1,2,\ldots,n-1}(z_1, z_2, \ldots, z_{n-1}) = (n-1)! f(z_1) f(z_2) \ldots f(z_{n-1})$$

if $\underline{c} \leq z_1 \leq z_2 \leq \cdots \leq z_{n-1} \leq \bar{c}$ and 0 otherwise. Furthermore, the density and the distribution function of the kth order statistic Z_k are

$$g_k(z_k) = \frac{(n-1)!}{(k-1)!\,(n-1-k)!} F(z_k)^{k-1} \left(1 - F(z_k)\right)^{n-1-k} f(z_k)$$

and

$$G_k(z_k) = \sum_{h=k}^{n-1} \binom{n-1}{h} F(z_k)^h \left(1 - F(z_k)\right)^{n-1-h}.$$

See, e.g., David and Nagaraja (2003) for a derivation of these results.

Some of the results we will obtain in the course of the paper require an additional assumption concerning the distribution function F. More precisely, we will sometimes assume the density f to be *logconcave.*[4] As shown by An (1998), logconcavity of $f(c)$ implies logconcavity of $F(c)$ as well as $1 - F(c)$ which in turn implies

$$\frac{d}{dc}\left(\frac{F(c)}{f(c)}\right) \geq 0 \qquad \text{and} \qquad \frac{d}{dc} E[C \mid C < c] \leq 1 \tag{1}$$

as well as

$$\frac{d}{dc}\left(\frac{1 - F(c)}{f(c)}\right) \leq 0 \qquad \text{and} \qquad \frac{d}{dc} E[C \mid C > c] \leq 1. \tag{2}$$

[4]There are many widely used distributions that have this property. Among them are the uniform distribution, the power distribution with an exponent > 1, the beta distribution with both parameters ≥ 1, the normal, exponential, extreme value, and logistic distribution. Of course, the last few distributions do not fit our model since they have infinite support. Note, however, that logconcavity is preserved when constructing a new distribution by truncating the support of one of those distributions. See Bagnoli and Bergstrom (2005) for a more detailed list and a proof of the truncation property.

Moreover, logconcavity of $F(c)$ and $1 - F(c)$ also results in logconcavity of $G_k(c)$ and $1 - G_k(c)$.

3 A Framework for Signaling in Auctions

In this section we analyze the bidding behavior in the auction conducted in the first stage in a fairly general framework. Most notably, we postpone the formulation of an explicit model of the product market in the second stage to the next section. Instead, we summarize the outcome of the second stage by two functions, π^W and π^L, that represent the profit a firm expects to earn at the beginning of the second stage, depending on whether it belongs to the winners or to the losers of the auction. We generally assume that $\pi^W > \pi^L$. Of course, these expected profits crucially depend on the beliefs firms hold about their rivals' costs.

As we have made mention of above, the auctioneer, after having received all the bids, publicly reveals a subset of them. In doing so, the auctioneer follows an announcement rule that specifies which bids are to be revealed depending on the order of the bids. For example, this rule could be to announce the highest bid. As we focus on separating equilibria, revealing bids is equivalent to revealing costs, since in equilibrium a firm's cost parameter can directly be inferred from its bid. Thus, through the auctioneer's announcement all firms learn the realization and the rank of a subset of all cost parameters. We denote this set of information about cost parameters by $\mathcal{I}$.

Having learnt $\mathcal{I}$ in the auction stage, firms update their beliefs concerning their competitors' cost parameters accordingly. As a result, firm i expects that the (ordered) vector of its rivals' costs is $\boldsymbol{\zeta}^i := E[\mathbf{Z}^i \,|\, \mathcal{I}]$. In addition, firm i's choice of action for the second stage also depends on how its type is perceived by the other firms. Let $\xi^i := E[C_i \,|\, \mathcal{I}]$ denote the cost parameter i's competitors believe firm i to have.[5]

With the above definitions at hand, let $\pi^W(c_i, \xi^i, \boldsymbol{\zeta}^i)$ denote the expected profit of firm i if it has won the auction. Aside from the realization of i's costs, i's expected profit also depends on the costs i's rivals expect it to have as well as the costs i expects its rivals to have. Similarly, let $\pi^L(c_i, \xi^i, \boldsymbol{\zeta}^i)$ denote the expected profits of firm i if it has lost the auction.

We assume that a higher cost parameter c_i always implies lower expected profits,

[5] Note that with this definition of beliefs we assume that beliefs depend solely on the commonly known information set $\mathcal{I}$ and not on any private information. In doing so we exclude some announcement rules from our framework, as we further discuss below in Subsection 3.3.

hence, $\frac{\partial}{\partial c_i}\pi^t(c_i, \xi^i, \boldsymbol{\zeta}^i) < 0$ for $t = W, L$. Further, we assume

$$\frac{\partial}{\partial c_i}\left(\pi^W(c_i, c_i, \mathbf{z}^i) - \pi^L(c_i, c_i, \mathbf{z}^i)\right) < 0,$$

i.e., under complete information, a low cost firm benefits more from winning the auction than a high cost firm. Therefore, in the absence of any signaling effects, we would expect a low cost firm to be willing to pay more for winning the auction. That is why we will in general look for a separating equilibrium where the firms with the lowest cost parameters win the auction.

3.1 A Direct Mechanism

For the derivation of equilibrium bidding in various auction formats, it is useful, as a first step, to analyze a corresponding direct mechanism where firms, instead of placing a bid, are asked to report their types to the auctioneer. In our setting, such a mechanism consists of three components: an allocation rule choosing the winners among firms, a payment rule specifying the amount each firm has to pay, and an announcement rule for the auctioneer. As for the allocation rule, we focus on the class of direct mechanisms that select the k firms with the lowest reported costs as winners. Regarding the payment rule we take the following notational shortcut. Instead of defining a function that fixes a payment for each firm depending on all reports, we simply let $\hat{m}(x)$ denote the expected payment by a firm that reports to be of type x while all its rivals report their true types. Let I denote the announcement rule where I is a function returning a subset of the reports depending on their order. From now on, we will refer to such a direct mechanism as $\langle \hat{m}, I \rangle$.

A direct mechanism that has an equilibrium where all firms choose to report their type truthfully is called *incentive compatible.* An incentive compatible direct mechanism $\langle \hat{m}, I \rangle$ must therefore ensure that no firm has an incentive to unilaterally deviate from the truth-telling equilibrium. Consider the point of view of firm i that reports type x while all other firms report their true type. Let $\mathbf{z}$ denote the realization of $\mathbf{Z}^i$. In this case, the auctioneer's announcement will depend on x and $\mathbf{z}$, such that we write the announcement as $I(x, \mathbf{z})$. The information set $\mathcal{I}$ contains the value of $I(x, \mathbf{z})$ combined with knowledge about the exact functional form of I. Consequently, given a specific announcement rule I, the beliefs relevant for firm i's expected profits are also functions of x and $\mathbf{z}$, such that we can write $\xi^i = \xi(x, \mathbf{z})$ and $\boldsymbol{\zeta}^i = \boldsymbol{\zeta}(x, \mathbf{z})$. Now, define the expected second stage profit of firm i conditional on the order statistic Z_k^i as

$$\Pi^t(c_i,x,z_k) := E\left[\pi^t(c,\xi(x,\mathbf{Z}^i),\boldsymbol{\zeta}(x,\mathbf{Z}^i))\,|\,Z_k^i = z_k\right]$$
$$= \int_{\underline{c}}^{z_k}\dots\int_{z_{k-2}}^{z_k}\int_{z_k}^{\overline{c}}\dots\int_{z_{n-2}}^{\overline{c}} \pi^t(c_i,\xi(x,\mathbf{z}),\boldsymbol{\zeta}(x,\mathbf{z}))\frac{g_{1,\dots,n-1}(\mathbf{z})}{g_k(z_k)}dz_{n-1}\dots dz_{k+1}dz_{k-1}\dots dz_1$$

for $t = W, L$. Since $\mathbf{Z}^i$ follows the same distribution for all i, we will from now on drop superscript i. When all other firms play according to the truth-telling equilibrium strategy, the expected payoff of firm i that has cost parameter c and reports to have cost parameter x is

$$\begin{aligned} U(c,x) &:= -\hat{m}(x) + (1-G_k(x))\,E\left[\pi^W(c,\xi(x,\mathbf{Z}),\boldsymbol{\zeta}(x,\mathbf{Z}))\,|\,Z_k > x\right] \\ &\qquad + G_k(x)E\left[\pi^L(c,\xi(x,\mathbf{Z}),\boldsymbol{\zeta}(x,\mathbf{Z}))\,|\,Z_k < x\right] \\ &= -\hat{m}(x) + \int_x^{\overline{c}}\Pi^W(c,x,z_k)g_k(z_k)dz_k + \int_{\underline{c}}^x \Pi^L(c,x,z_k)g_k(z_k)dz_k. \end{aligned} \tag{3}$$

The following lemma identifies incentive compatible direct mechanisms.[6]

Lemma 1 *The direct mechanism $\langle\hat{m}, I\rangle$ is incentive compatible if,* $\forall c,x\in[\underline{c},\overline{c}]$,

$$\begin{aligned} \hat{m}(c) &= \hat{m}(\overline{c}) + \int_c^{\overline{c}}\left(\Pi^W(y,y,y) - \Pi^L(y,y,y)\right)g_k(y)dy \\ &\quad - \int_c^{\overline{c}}\int_y^{\overline{c}}\Pi_2^{W\prime}(y,y,z_k)g_k(z_k)dz_kdy - \int_c^{\overline{c}}\int_{\underline{c}}^y \Pi_2^{L\prime}(y,y,z_k)g_k(z_k)dz_kdy \end{aligned} \tag{IC1}$$

and

$$\begin{aligned} U_{12}''(c,x) &= -\left(\Pi_1^{W\prime}(c,x,x) - \Pi_1^{L\prime}(c,x,x)\right)g_k(x) \\ &\quad + \int_x^{\overline{c}}\Pi_{12}^{W\prime\prime}(c,x,z_k)g_k(z_k)dz_k + \int_{\underline{c}}^x\Pi_{12}^{L\prime\prime}(c,x,z_k)g_k(z_k)dz_k \geq 0. \end{aligned} \tag{IC2}$$

Moreover, if $\langle\hat{m}, I\rangle$ is incentive compatible, then (IC1) *holds and* $U_{12}''(c,c)\geq 0$.

Proof. The the direct mechanism $\langle\hat{m}, I\rangle$ is incentive compatible iff

$$c\in\arg\max_x U(c,x) \qquad \forall c\in[\underline{c},\overline{c}]\,. \tag{4}$$

[6] For a function H of multiple variables, we use H_i' to denote the partial derivative with respect to the ith argument. Similarly, H_{ij}'' denotes the mixed partial derivative with respect to the ith and jth argument.

Sufficient for (4) is the first order condition

$$U_2'(c,c) = 0 \tag{5}$$

together with the condition that

$$U_2'(c,x) \geq (\leq)\, 0 \qquad \forall x < (>)\, c; c, x \in [\underline{c}, \overline{c}]\,. \tag{6}$$

Integrating (5) from c to $\overline{c}$ on both sides and rearranging, we obtain (IC1). Because of (5) we have

$$U_2'(c,x) = \int_x^c U_{12}''(y,x)dy.$$

Hence, $U_{12}''(c,x) \geq 0 \;\; \forall c, x \in [\underline{c}, \overline{c}]$ is sufficient for (6) which is stated in (IC2).

On the other hand, (5) and $U_{22}''(c,c) \leq 0$ are necessary for (4). Taking the derivative of (5) on both sides, we receive $U_{12}''(c,c) + U_{22}''(c,c) = 0$ such that $U_{22}''(c,c) \leq 0$ is equivalent to $U_{12}''(c,c) \geq 0$. ■

In general, it does not seem plausible to assume that firms can be forced to take part in the auction stage. Hence, we are interested in direct mechanisms where firms voluntarily choose to participate. Such mechanisms are often referred to as being *individually rational*. A mechanism is individually rational for a firm if its expected equilibrium payoff $U(c,c)$ is higher than the payoff it would earn when not participating. In our model, the value of a firm's outside option is simply its expected profit in the second stage without having access to the cost reducing technology. Hence, in contrast to standard models in mechanism design theory, firm i's outside option does crucially depend on the beliefs firms hold about the types of their competitors when they observe that firm i is not participating. Since not participating lies outside the equilibrium path, beliefs concerning this event are not restricted by a perfect Bayesian equilibrium. As the following lemma shows, sufficient conditions for a mechanism to be individually rational are that the expected payment by a firm with cost parameter $\overline{c}$ is zero and that a firm that refuses to participate is perceived by the other firms as being of type $\overline{c}$.

Lemma 2 *The incentive compatible direct mechanism $\langle \hat{m}, I \rangle$ is individually rational if $\hat{m}(\overline{c}) = 0$ and (out-of-equilibrium) beliefs are*

$$\begin{aligned} E[C_i \,|\, i \text{ does not participate} \cup \mathcal{I}] &= \xi(\overline{c}, \mathbf{z}), \\ E[\mathbf{Z}^i \,|\, i \text{ does not participate} \cup \mathcal{I}] &= \boldsymbol{\zeta}(\overline{c}, \mathbf{z}). \end{aligned} \tag{7}$$

Proof. With beliefs (7) and $\hat{m}(\overline{c}) = 0$, the expected payoff of a firm with costs $\overline{c}$ is the same regardless whether it participates or not. Therefore, a firm with costs $\bar{c}$ will not refuse to participate. All other types of firms could, by reporting $\bar{c}$ instead of their true type, achieve the same payoff as if they did not participate. But since the mechanism is incentive compatible, they are better off participating and reporting truthfully. ■

As we will frequently refer to incentive compatible and individually rational direct mechanisms throughout this paper, it is useful to simplify the exposition by defining

$$m(c) := \int_c^{\bar{c}} \left(\Pi^W(y,y,y) - \Pi^L(y,y,y)\right) g_k(y)dy \\ - \int_c^{\bar{c}} \int_y^{\bar{c}} \Pi_2^{W\prime}(y,y,z_k) g_k(z_k) dz_k dy - \int_c^{\bar{c}} \int_{\underline{c}}^{y} \Pi_2^{L\prime}(y,y,z_k) g_k(z_k) dz_k dy. \quad (8)$$

Observe that by Lemmata 1 and 2, the direct mechanism $\langle m, I\rangle$ is incentive compatible if (IC2) while it is individually rational under beliefs (7).

3.2 Equilibrium Bidding in some Standard Auctions

In the following, we derive the equilibrium bidding strategies for three well-known auction formats: the all-pay auction, the discriminatory auction, and the uniform-price auction. In all three auctions access to the new technology is awarded to the k highest bidders. The auctions differ, however, in terms of their payment rules. In the all-pay auction, each firm has to pay its bid, regardless whether it has won or lost. In both of the other formats, losers do not pay anything. The winners of a discriminatory auction have to pay their bid, whereas in the uniform-price auction the winners all must pay the highest losing bid.

In a separating equilibrium, firms bid according to a strictly monotone strategy $\beta : [\underline{c}, \bar{c}] \to \mathbb{R}_+$. Suppose an auction has a separating equilibrium with a strictly decreasing β. Thanks to the revelation principle, such an auction is equivalent to an incentive compatible direct mechanism $\langle \hat{m}, I\rangle$. Therefore, we can easily derive such equilibrium strategies for the three auctions by making use of the results of the preceding subsection.

Proposition 1 *Consider $m(c)$ as defined in* (8) *and suppose firms' out-of-equilibrium beliefs are represented by* (7)*. Define*

$$\beta_A(c) := m(c)\ , \qquad \beta_D(c) := \frac{m(c)}{1 - G_k(c)}\ , \qquad \textit{and} \qquad \beta_U(c) := -\frac{m'(c)}{g_k(c)} \quad (9)$$

where β_A, β_D, and β_U denote bidding strategies for the all-pay, the discriminatory, and the uniform-price auction. For $T = A, D, U$ holds the following result. If (IC2) *and*

$\beta'_T(c) < 0$, there exists an individually rational separating equilibrium of the auction format T where a firm with cost parameter c bids the amount $\beta_T(c)$. Provided the auctioneer uses the same announcement rule I, all three auction formats are revenue equivalent.

Proof. First note that for a separating equilibrium to exist, bidding strategies have to be strictly monotone so that firms can infer types from revealed bids. If equilibrium bidding strategies are strictly decreasing, all three auction formats choose the firms with the lowest costs as the winners. Since type $\bar{c}$ never wins the auction, its expected payment in the discriminatory and the uniform-price auction is zero. In order to be sure that individual rationality is fulfilled also in the all-pay auction, we consider only the equilibrium where type $\bar{c}$ bids zero. Hence, given an announcement rule I, all three auctions are equivalent to an incentive compatible direct mechanism $\langle \hat{m}, I \rangle$ with $\hat{m}(\bar{c}) = 0$. Incentive compatibility implies (IC1) such that we have $\hat{m}(c) = m(c)$. Together with (IC2) this is also sufficient for incentive compatibility. The expected payments by the firms and therefore also the expected revenue for the auctioneer are the same in all three auctions. Expected payment of type c has to equal $m(c)$ in all three auction formats, i.e.

$$m(c) = \beta_A(c) = (1 - G_k(c))\, \beta_D(c) = \int_c^{\bar{c}} \beta_U(y) g_k(y) dc.$$

This can be rearranged to yield (9). With beliefs (7), all three auctions are individually rational. ■

There are a few things worth noting concerning the uniqueness of equilibrium strategies. First, since losers do not pay anything in the discriminatory and uniform-price auction, we must have $\hat{m}(\bar{c}) = 0$ in those two cases. Hence, the incentive compatible $\hat{m}$ and therefore also the strictly decreasing equilibrium strategies β_D and β_U are unique. Furthermore, under beliefs (7) the discriminatory and uniform-price auction are individually rational. On the other hand, for the all-pay auction, $\hat{m}(\bar{c})$ is not necessarily zero and hence the equilibrium strategy is not unique. Yet with out-of-equilibrium beliefs (7) the equilibrium strategy β_A defined in Proposition 1 corresponds to the unique individually rational equilibrium of the all-pay auction.

According to Proposition 1, two conditions have to be fulfilled in order for a separating equilibrium to exist for a specific auction format. First, (IC2) must hold to ensure incentive compatibility of the corresponding direct mechanism. Second, the equilibrium bidding strategy needs to be strictly decreasing so that types can be inferred from bids. While the first condition is, of course, the same for all auction formats, the second condition differs.

Corollary 1 *Given the same announcement rule is used in all three auction formats,*

$$\beta_U'(c) < 0 \;\Rightarrow\; \beta_D'(c) < 0 \;\Rightarrow\; \beta_A'(c) < 0.$$

Consequently, if the uniform-price auction has a separating equilibrium, the same is true for the discriminatory auction which in turn implies that the all-pay auction has a separating equilibrium.

Proof. Note that $\beta_D(c) = \frac{\int_c^{\bar{c}} \beta_U(z_k) g_k(z_k) dz_k}{1-G_k(c)} = E\left[\beta_U(Z_k) \mid Z_k > c\right]$. Therefore $\beta_U'(c) < 0$ implies $\beta_D'(c) < 0$. Since $\frac{d}{dc}\left(1 - G_k(c)\right) < 0$, $\beta_D'(c) < 0$ implies $\beta_A'(c) < 0$. ■

This very useful result suggests that there can, e.g., be situations where the all-pay auction has a separating equilibrium while the other two auction formats do not. Since $\beta_D(c) = E\left[\beta_U(Z_k) \mid Z_k > c\right]$, the discriminatory and the uniform-price auction are closely related. Note, however, that $\beta_D'(c) < 0$ only implies $\beta_U'(c) \leq 0$ so that existence of a separating equilibrium in the discriminatory auction does in general not imply existence of a separating equilibrium in the uniform-price auction.

3.3 Announcement Rules

So far, we have not specified what rule the auctioneer follows when announcing a subset of the bids. We have just assumed this announcement to affect the firms' beliefs through revealing information $\mathcal{I}$ about cost parameters which we have described by an announcement rule I for the corresponding direct mechanism. Of course, there are many possibilities when choosing a bid announcement policy. In this paper, we focus on the cases where either all bids are revealed or where the amount each bidder must pay is announced. For the auction formats we examine, this corresponds to three different announcement rules I.

Suppose the auctioneer publicly reveals *all bids* so that, in a separating equilibrium, the cost parameters of all firms become commonly known. We denote this announcement rule by I^{ab}. Beliefs hence simply become

$$\xi^{ab}(x, \mathbf{z}) = x \qquad \text{and} \qquad \boldsymbol{\zeta}^{ab}(x, \mathbf{z}) = \mathbf{z}, \tag{10}$$

so that firms act under complete information in the second stage.

Let us turn to the case where the prices paid by the bidders become publicly known after the auction. Of course, this makes no difference for the all-pay auction. For the discriminatory auction, however, revealing the prices paid means that the auctioneer announces the *winning bids* only. In a separating equilibrium with decreasing bidding strategies, the k lowest costs become common knowledge which we denote by I^{wb}. Concerning the rest of the cost parameters firms merely know that they are higher than the kth lowest cost parameter. Thus, beliefs are

$$\xi^{wb}(x, \mathbf{z}) = \begin{cases} x & \text{for} \quad x < z_k \\ E\left[C_i \,|C_i > z_k\right] & \text{for} \quad x > z_k \end{cases} \tag{11}$$

and

$$\boldsymbol{\zeta}^{wb}(x, \mathbf{z}) = \begin{cases} E\left[\mathbf{Z} \,|\, Z_1 = z_1, Z_2 = z_2, ..., Z_{k-1} = z_{k-1}\right] & \text{for} \quad x < z_{k-1} \\ E\left[\mathbf{Z} \,|\, Z_1 = z_1, Z_2 = z_2, ..., Z_{k-1} = z_{k-1}, Z_k > x\right] & \text{for} \quad z_{k-1} < x < z_k \\ E\left[\mathbf{Z} \,|\, Z_1 = z_1, Z_2 = z_2, ..., Z_k = z_k\right] & \text{for} \quad x > z_k. \end{cases} \tag{12}$$

Revealing the prices paid in a uniform-price auction corresponds to the announcement rule where only the *highest losing bid* is revealed. Consequently, in an equilibrium in decreasing strategies, only the $(k+1)$th lowest cost parameter becomes publicly known which we denote by I^{hlb}. Recalling we assumed that firms all know the identities of the winners, we obtain for the beliefs under rule I^{hlb}

$$\xi^{hlb}(x, \mathbf{z}) = \begin{cases} E[C_i \,|\, C_i < z_k] & \text{for} \quad x < z_k \\ x & \text{for} \quad z_k < x < z_{k+1} \\ E[C_i \,|\, C_i > z_{k+1}] & \text{for} \quad x > z_{k+1} \end{cases} \tag{13}$$

and

$$\boldsymbol{\zeta}^{hlb}(x, \mathbf{z}) = \begin{cases} E[\mathbf{Z} \,|\, Z_k = z_k] & \text{for} \quad x < z_k \\ E[\mathbf{Z} \,|\, Z_k < x < Z_{k+1}] & \text{for} \quad z_k < x < z_{k+1} \\ E[\mathbf{Z} \,|\, Z_{k+1} = z_{k+1}] & \text{for} \quad x > z_{k+1}. \end{cases} \tag{14}$$

The three announcement rules we have just defined all have a fundamental property in common: the revealed information allows a firm to determine whether it belongs to the winners or the losers of the auction.[7] Most importantly, this property implies that beliefs are independent of any private information. Hence, two firms i and j will hold exactly the same beliefs concerning the costs c_h for all $h \neq i, j$ when they enter the second stage. One reason why we restrict our analysis to announcement rules that exhibit this property is that it allows for a closed form solution to our model of the second stage we present in Section 4. As an example where this property does not hold, consider the situation that arises when the auctioneer does not announce any bids at all. Of course, also in this

[7] This is the case if at least either the kth or the $(k+1)$th lowest cost parameter becomes publicly known.

case, we would still want to assume that each firm learns whether it has won access to the new technology. If the truth-telling firm i has won (lost), it will form beliefs about its competitors conditional on $Z_k > c_i$ ($Z_k < c_i$). Therefore, firms will hold differing beliefs about their competitors, as for each firm i beliefs ξ^i and $\boldsymbol{\zeta}^i$ depend on its privately known cost parameter c_i.

3.4 The Signaling Effect

In order to analyze how signaling affects the equilibrium behavior of firms, it is useful to compare our results to the case where signaling is not possible. Suppose, as a benchmark case, that all cost parameters are directly revealed to firms at the beginning of the second stage. In this case, the type x firm i might pretend to be has no effect on Π^W and Π^L. Using (8), the expected payment by a firm in this benchmark case simplifies to

$$m^b(c) := \int_c^{\bar{c}} \left(\Pi^W(y,y,y) - \Pi^L(y,y,y)\right) g_k(y)dy.$$

This allows us to decompose the expected payment into a non-signaling and a signaling part, i.e.,

$$m(c) = m^b(c) + m^s(c)$$

where the signaling component is given by

$$m^s(c) := -\int_c^{\bar{c}} \int_y^{\bar{c}} \Pi_2^{W\prime}(y,y,z_k)g_k(z_k)dz_k dy - \int_c^{\bar{c}} \int_{\underline{c}}^{y} \Pi_2^{L\prime}(y,y,z_k)g_k(z_k)dz_k dy.$$

While $m^b(c)$ is clearly positive, the sign of the signaling effect $m^s(c)$ depends on the model of the second stage. Depending on the kind of interaction among firms after the auction, signaling might increase or decrease bids and expected payments. Note that we will drop the superscripts W and L in the following. The direction of the signaling effect depends, of course, on the sign of $\Pi_2'(c,x,z_k) = E\left[\frac{\partial}{\partial x}\pi(c,\xi(x,\mathbf{Z}),\boldsymbol{\zeta}(x,\mathbf{Z})) \mid Z_k = z_k\right]$, i.e., on the sign of

$$\frac{\partial}{\partial x}\pi(c,\xi(x,\mathbf{z}),\boldsymbol{\zeta}(x,\mathbf{z})) = \frac{\partial \pi(c,\xi,\boldsymbol{\zeta})}{\partial \xi}\frac{\partial \xi(x,\mathbf{z})}{\partial x} + \sum_{j=1}^{n-1} \frac{\partial \pi(c,\xi,\boldsymbol{\zeta})}{\partial \zeta_j}\frac{\partial \zeta_j(x,\mathbf{z})}{\partial x}.$$

Hence, there are two effects through which signaling has an impact on firm i's behavior. The first effect stems from the way firm i wants to be perceived by its competitors, i.e. $\frac{\partial \pi(c,\xi,\boldsymbol{\zeta})}{\partial \xi}$. The second effect is due to $\frac{\partial \pi(c,\xi,\boldsymbol{\zeta})}{\partial \zeta_j}$, the influence of the expected cost parameters of firm i's competitors on expected profits. The strength of those effects depends on how the signal firm i sends is reflected in the beliefs, i.e., on the announcement rule.

As we have seen in Subsection 3.3, if the auctioneer announces all bids, we have $\frac{\partial \xi(x,\mathbf{z})}{\partial x} = 1$ and $\frac{\partial \zeta_j(x,\mathbf{z})}{\partial x} = 0$ for all j. In this case, there is only the first effect. If $\frac{\partial \pi(c,\xi,\boldsymbol{\zeta})}{\partial \xi} > 0$, as, e.g., in a Bertrand oligopoly, firm i prefers to be thought of as having high costs. The signaling effect therefore reduces the bids and expected payments of firms. On the other hand, if $\frac{\partial \pi(c,\xi,\boldsymbol{\zeta})}{\partial \xi} < 0$, as, e.g., in a Cournot oligopoly, firm i wants to pretend to have lower costs than it actually has, such that the signaling effect increases bids and payments.

Looking at the other two announcement rules we consider in Subsection 3.3, we find that in both cases $\frac{\partial \xi(x,\mathbf{z})}{\partial x} > 0$ and, for some j, $\frac{\partial \zeta_j(x,\mathbf{z})}{\partial x} > 0$ for a certain range of x. If firm i's bid is actually announced by the auctioneer, this will not only affect ξ but also $\boldsymbol{\zeta}$. Consider, e.g., a discriminatory auction with the winning bids being announced. In the event that firm i pretending to be of type x just wins with the kth highest bid, it will be generally believed that the losers must have cost parameters higher than x. This is how the second effect comes into play. Its direction depends on how i's profit depends on the cost parameters of i's competitors. If $\frac{\partial \pi(c,\xi,\boldsymbol{\zeta})}{\partial \zeta_j} > 0$, as in an oligopoly market where goods are substitutes, the second effect reduces bids and payments. By contrast, if $\frac{\partial \pi(c,\xi,\boldsymbol{\zeta})}{\partial \zeta_j} < 0$, as in an oligopoly with firms producing complements, bids and payments are increased.

3.5 Relation to the Literature

Before moving on to the next section where we develop our model for the market interaction in the second stage, let us digress for a moment in order to demonstrate how our framework accommodates several interesting examples from the literature.

3.5.1 Katzman and Rhodes-Kropf (2008)

Katzman and Rhodes-Kropf (2008) construct a model where n firms with privately known marginal costs c_i bid in an auction in order to win access to a duopoly with an incumbent monopolist. Hence, in our framework, we have $k = 1$ and $\pi^L(c,\xi,\boldsymbol{\zeta}) = 0$. Moreover, the winner's expected duopoly profit is independent of the other bidders' types, i.e. $\pi^W(c,\xi,\boldsymbol{\zeta}) = \hat{\pi}(c,\xi)$. The authors compare a first-price and second-price auction where the winner's bid is revealed to an English auction where the second highest bid is revealed. In our terminology the equilibrium of these auctions corresponds to the strategies β_D^{wb}, β_U^{wb}, and β_U^{hlb}.

Note that when the winning bid is announced, we have $\Pi^W(c,x,z_1) = \hat{\pi}(c,x)$ if $x < z_1$.

Therefore, (IC1) simplifies to

$$\begin{aligned}\hat{m}(c) &= \hat{m}(\bar{c}) + \int_c^{\bar{c}} \hat{\pi}(y,y) g_1(y) dy - \int_c^{\bar{c}} \int_y^{\bar{c}} \hat{\pi}_2'(y,y) g_1(z_1) dz_1 dy \\ &= \hat{m}(c^*) + \int_c^{c^*} \left\{ \hat{\pi}(y,y) g_1(y) - \hat{\pi}_2'(y,y) \left(1 - G_1(y)\right) \right\} dy\end{aligned}$$

where $c^* \in (\underline{c}, \bar{c})$. Katzman and Rhodes-Kropf (2008) endogenize bidder participation where c^* is the highest cost type that participates. With $\hat{m}(c^*) = 0$ instead of $\hat{m}(\bar{c}) = 0$ we can apply Proposition 1 in order to find

$$\beta_U^{wb}(c) = \hat{\pi}(c,c) - \hat{\pi}_2'(c,c) \frac{1 - G_1(c)}{g_1(c)}.$$

Since $1 - G_1(z_1) = (1 - F(z_1))^{n-1}$, this is exactly the second-price auction equilibrium Katzman and Rhodes-Kropf (2008) find (Theorem 3, p. 68). Their first-price auction equilibrium can be obtained in a very similar way.

When the second highest bid is revealed, we have $\Pi^W(c, x, z_1) = \hat{\pi}(c, E[C_i \,|\, C_i < z_1])$ if $x < z_1$. Therefore,

$$\beta_U^{hlb}(c) = \hat{\pi}(c, E[C_i \,|\, C_i < c])$$

which is exactly the English auction equilibrium of Katzman and Rhodes-Kropf (2008) (see Theorem 4 on p. 70).

3.5.2 Das Varma (2003) and Goeree (2003)

Both Das Varma (2003) and Goeree (2003) develop a model of bidding in a first-price auction through which single access ($k = 1$) to a cost reducing innovation is sold. Following the auction, the winning bid is disclosed. Bidding in this auction are n firms that compete in a market afterwards. The actual value of the the cost reduction to each firm is private information while all other parameters of the model are commonly known.

Since only the type of the winner is relevant for the second stage, we have $\pi^W(c, \xi, \boldsymbol{\zeta}) = \tilde{\pi}^W(c, \xi)$ and $\pi^L(c, \xi, \boldsymbol{\zeta}) = \tilde{\pi}^L(\zeta_1)$. The winning bid being revealed in turn implies $\Pi^W(c, x, z_1) = \tilde{\pi}^W(c, x)$ as well as $\Pi^L(c, x, z_k) = \tilde{\pi}^L(z_1)$. From Proposition 1 we have

$$\beta_D^{wb}(c) = \frac{1}{1 - G_1(c)} \int_c^{\bar{c}} \left\{ \left(\tilde{\pi}^W(y,y) - \tilde{\pi}^L(y) \right) g_1(y) - \tilde{\pi}_2^{W\prime}(y,y) \left(1 - G_1(y)\right) \right\} dy$$

Note that if $c = s - \theta$ where $\theta \sim \tilde{F}$ on $\left[\underline{\theta}, \overline{\theta}\right]$, we have $F(c) = 1 - \tilde{F}(s - c)$. Using the fact that $1 - G_1(c) = \tilde{F}(s-c)^{n-1}$ and performing a change of variable we obtain

$$\beta_D^{wb}(\theta) = \int_{\underline{\theta}}^{\theta} \left\{ \tilde{\pi}^W(s-t, s-t) - \tilde{\pi}^L(s-t) \right.$$
$$\left. - \tilde{\pi}_2^{W\prime}(s-t, s-t) \frac{\tilde{F}(t)}{(n-1)\,\tilde{f}(t)} \right\} \frac{(n-1)\,\tilde{F}(t)^{n-2}\tilde{f}(t)}{\tilde{F}(\theta)^{n-1}} dt.$$

Taking their slightly different definition of $\tilde{\pi}^W$ and $\tilde{\pi}^L$ into account, this is exactly the first-price auction equilibrium Das Varma (2003) and Goeree (2003) find (see Proposition 2 on p. 28 and Proposition 4 on p. 356, respectively).

In addition, Goeree (2003) also looks at two other auction formats. While his second-price auction fits our framework very well, the English auction he considers does not. The reason for this is that Goeree (2003), in contrast to Katzman and Rhodes-Kropf (2008), assumes the winning bid of the English auction to be revealed which induces the equilibrium strategy to differ from the ones we consider in Proposition 1.

3.5.3 Ziv (1993)

Ziv (1993) analyzes a model of $n = 2$ firms with privately known marginal costs c_i and c_j competing in a Cournot duopoly. Before playing the Cournot game both firms signal their type through publicly burning money. In our framework this corresponds to the all-pay auction with all bids revealed. Furthermore, we have $\pi^W(c_i, \xi^{ab}, \boldsymbol{\zeta}^{ab}) = \pi^L(c_i, \xi^{ab}, \boldsymbol{\zeta}^{ab}) = \pi(c_i, x, c_j)$ such that

$$\beta_A^{ab}(c_i) = -\int_{c_i}^{\overline{c}} E\left[\pi_2'(y, y, C_j)\right] dy$$

which corresponds to the equilibrium strategy (13) in Ziv (1993) when substituting the Cournot profit for π and engaging in some rearranging.

An additional example that fits our framework is the simultaneous auction model of Katsenos (2007) where $k = 2$ licenses granting access to an oligopoly market are sold to $n > 2$ firms through a discriminatory auction. With the winning bids being revealed, firms are using the auction to signal about their privately known marginal costs.

Apart from Ziv (1993), in all the models we have discussed above only the private information of the winners of the auction is relevant for the second stage. Of course, in Ziv (1993) all private information is relevant, but since nothing can be won in the auction there is no distinction between winners and losers. In contrast to that, our model exhibits both of these features: all private information is relevant and the auction stage is used to sell objects with an actual intrinsic value. Moreover, our framework allows for analyzing multi-object auctions, whereas Katzman and Rhodes-Kropf (2008), Das Varma (2003), and Goeree (2003) focus on $k = 1$.

4 The Second Stage

After having kept the second stage quite general when developing the framework for the auction stage, we now describe a specific model of the market competition in the second stage. Having explicit solutions for π^W and π^L at hand enables us to explore under what circumstances separating equilibria arise in auctions among competitors.

The production technology of the firms exhibits constant marginal costs and no fixed costs. Marginal costs differ among firms and are private information. For firm i, marginal costs are described by the cost parameter c_i. The technological innovation sold through the auction reduces marginal costs by a constant amount ε.

We will consider two forms of competition: either firms choose quantities simultaneously (Cournot competition) or they set prices simultaneously (Bertrand competition). In both cases each firm faces a linear demand for its product. The inverse demand is given by

$$p_i = a - q_i - d \sum_{j \neq i} q_j \tag{15}$$

where $d \in (0, 1]$ and where p_i and q_i denote the price and the quantity of firm i's product. The parameter d captures the degree of differentiation between the products of the firms. In particular, if $d = 1$, firms all produce a homogeneous good and if $d \to 0$, all firms would become monopolists. We generally assume

$$a > \tilde{a} := \overline{c} + (n-1)\left(\overline{c} - \underline{c} + \varepsilon\right). \tag{16}$$

Moreover, in the case of Bertrand competition we make the additional assumption

$$d < \frac{2a - \overline{c}}{2a - \overline{c} + \tilde{a}}. \tag{17}$$

These restrictions on parameters guarantee that all firms will supply a positive amount of their product in the second stage, regardless of the realization of marginal costs and of the allocation of the cost reducing technology.

Consider first a situation where no firm has access to the cost reducing technology. Furthermore, assume that firms all have learned the same information $\mathcal{I}$ about the realization of all marginal costs. In the case of Cournot competition, each firm i chooses its production quantity q_i in order to maximize its expected profits. Let $q_i : [\underline{c}, \overline{c}] \to \mathbb{R}_+$ denote the equilibrium strategy of firm i. In the Bayesian Nash equilibrium we must have

$$q_i(c_i) = \arg\max_q \left(a - q - d \sum_{j \neq i} E[q_j(C_j) \,|\, \mathcal{I}] - c_i \right) q \quad \forall i \text{ and } c_i \in [\underline{c}, \overline{c}]\,. \tag{18}$$

Solving for this equilibrium and distinguishing between a firm i that has won and a firm i that has lost in the first stage, we find the profit firm i expects to earn.[8]

Lemma 3 *Under Cournot competition the expected profit of firm i that has won and lost the auction, respectively, is*

$$\pi^W(c_i, \xi, \boldsymbol{\zeta}) = \left(\gamma_0 + \gamma_1 \sum_{j=1}^{n-1} \zeta_j - \gamma_2 \xi - \frac{1}{2} c_i + \left\{ \frac{1}{2} + \gamma_2 - (k-1)\gamma_1 \right\} \varepsilon \right)^2$$

and

$$\pi^L(c_i, \xi, \boldsymbol{\zeta}) = \left(\gamma_0 + \gamma_1 \sum_{j=1}^{n-1} \zeta_j - \gamma_2 \xi - \frac{1}{2} c_i - k\gamma_1 \varepsilon \right)^2$$

where $\gamma_0 := \frac{(4-2d)a}{2(4-d^2(n-1)+2d(n-2))}$, $\gamma_1 := \frac{2d}{2(4-d^2(n-1)+2d(n-2))}$, *and* $\gamma_2 := \frac{d^2(n-1)}{2(4-d^2(n-1)+2d(n-2))}$.

Proof. See Appendix A.1. ■

A property of expected profits important for our analysis is of course the signaling incentive it provides for a firm. Since $\pi_2^{t\prime}(c_i, \xi, \boldsymbol{\zeta}) = -2\gamma_2 \sqrt{\pi^t(c_i, \xi, \boldsymbol{\zeta})} < 0$ for $t = W, L$, a firm would like to pretend to be stronger than it actually is. That way, this firm can induce its competitors to supply a lower quantity and therefore obtain a higher market share.

Let us turn to the case of Bertrand competition. Here, each firm chooses the price p_i of its product in order to maximize its expected profit. Let $Q_i\left(\sum p_j\right)$ denote the demand for the product of firm i that corresponds to the inverse demand (15). Denoting firm i's equilibrium strategy by $p_i : [\underline{c}, \overline{c}] \to \mathbb{R}_+$, the Bayesian Nash equilibrium requires

$$p_i(c_i) = \arg\max_p \left\{ E\left[Q_i\left(p + \textstyle\sum_{j \neq i} p_j(C_j) \right) \middle| \mathcal{I} \right] - p \right\} (p - c_i) \qquad \forall i \text{ and } c_i \in [\underline{c}, \overline{c}]\,. \tag{19}$$

This implies the following for the expected profit of firm i.

[8] For two firms with $\varepsilon = 0$ and $d = 1$, this result is identical to (4) in Ziv (1993).

Lemma 4 *Under Bertrand competition the expected profit of firm i that has won and lost the auction, respectively, is*

$$\pi^W(c_i, \xi, \boldsymbol{\zeta}) = \left(\delta_0 + \delta_1 \sum_{j=1}^{n-1} \zeta_j + \delta_2 \xi - \frac{1}{2} c_i + \left\{\frac{1}{2} - \delta_2 - (k-1)\,\delta_1\right\} \varepsilon\right)^2$$

and

$$\pi^L(c_i, \xi, \boldsymbol{\zeta}) = \left(\delta_0 + \delta_1 \sum_{j=1}^{n-1} \zeta_j + \delta_2 \xi - \frac{1}{2} c_i - k\delta_1 \varepsilon\right)^2$$

where $\delta_0 := \frac{(1-d)a}{2(1-d)+d(n-1)}$, $\delta_1 := \frac{2d(1-2d+dn)}{2(2-3d+2dn)(2-3d+dn)}$, *and* $\delta_2 := \frac{d^2(n-1)}{2(2-3d+2dn)(2-3d+dn)}$.

Proof. See Appendix A.2. ■

Comparing Lemmata 3 and 4, we find the structure of expected profits to be very similar. The crucial difference lies in the signaling incentive. In the Bertrand market, as $\pi_2^{t\prime}(c_i, \xi, \boldsymbol{\zeta}) = 2\delta_2 \sqrt{\pi^t(c_i, \xi, \boldsymbol{\zeta})} > 0$ for $t = W, L$, a firm prefers to appear weaker than it is, so that its competitors set higher prices leaving a higher market share for the firm in question.

5 Revealing All Bids

Having presented our model for the second stage in the preceding section, we are now ready to study the full model. We begin, in this section, with the case where the auctioneer announces all bids at the end of the first stage. In the subsequent section we will then turn to the situation where the amount to be paid by each bidder is revealed.

Suppose the auctioneer reveals all bids after the auction, so that firms will, in equilibrium, have full information about all cost parameters when they enter the second stage. Consequently, with firms holding beliefs ξ^{ab} and $\boldsymbol{\zeta}^{ab}$ defined in (10), we have

$$\Pi^t(c, x, z_k) = E\left[\pi^t(c, x, \mathbf{Z}) \mid Z_k = z_k\right] \qquad \text{for } t = W, L. \tag{20}$$

Using Lemmata 3 and 4 together with Lemma 1, we find the following.

Proposition 2 *Under Cournot competition, the direct mechanism $\langle m, I^{ab} \rangle$ is incentive compatible. Under Bertrand competition, $\langle m, I^{ab} \rangle$ is incentive compatible iff $n = 2$ and*

$$\left\{\frac{1}{2} - \delta_2 + \delta_1\right\} \varepsilon f(c) - \delta_2 \geq 0 \qquad \text{for all } c \in [\underline{c}, \overline{c}]. \tag{21}$$

Proof. Under Cournot competition, we have

$$\begin{aligned}\Pi_1^{W\prime}(c,x,x)-\Pi_1^{L\prime}(c,x,x) &= E\left[\pi_1^{W\prime}(c,x,\mathbf{Z})-\pi_1^{L\prime}(c,x,\mathbf{Z})\,|\,Z_k=x\right]\\ &= -\left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon\end{aligned}$$

and

$$\Pi_{12}^{t\prime\prime}(c,x,z_k)=\gamma_2 \qquad \text{for } t=W,L.$$

(IC2) therefore simplifies to

$$\left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon g_k(x)+\gamma_2\geq 0 \tag{22}$$

implying that the direct mechanism with expected payment m defined by (8) is incentive compatible.

For the case of Bertrand competition, we can simply use (22) and replace γ_1 by δ_1 and γ_2 by $-\delta_2$. Hence, we obtain

$$U_{12}''(c,x)=\left\{\frac{1}{2}-\delta_2+\delta_1\right\}\varepsilon g_k(x)-\delta_2.$$

Now note that, $g_k(\underline{c})=0$ for $k>1$ and $g_k(\overline{c})=0$ for $k<n-1$. Consequently, if $n>2$, there are always some c close enough to $\underline{c}$ or $\overline{c}$ (or both) for which $U_{12}''(c,c)$ is strictly negative. Hence, $n=2$ is necessary for the direct mechanism to be incentive compatible. Incentive compatibility thus requires (21). As $U_{12}''(c,x)\geq 0\ \forall c,x$ is equivalent to $U_{12}''(c,c)\geq 0\ \forall c$, (21) and $n=2$ are also sufficient for incentive compatibility. ■

Intuitively, in auctions where the highest bidders win and without signaling possibilities, we would expect the firms with the lowest costs to bid the highest amount since $\frac{\partial}{\partial c}\left(\pi^W(c,\xi,\boldsymbol{\zeta})-\pi^L(c,\xi,\boldsymbol{\zeta})\right)<0$ for both the Cournot and the Bertrand model. Taking signaling into account, firms want to appear stronger under Cournot competition where $\pi_2'(c,\xi,\boldsymbol{\zeta})<0$, while they pretend to have high costs in the Bertrand case because of $\pi_2'(c,\xi,\boldsymbol{\zeta})>0$. Since under Cournot (Bertrand) competition we have $\pi_{12}''(c,\xi,\boldsymbol{\zeta})>0$ ($\pi_{12}''(c,\xi,\boldsymbol{\zeta})<0$), the signaling incentive is in both cases strongest for the low-cost types. In the Cournot case, the signaling effect thus goes into the same direction as the first effect: low-cost firms increase their already higher bids by more than high-cost firms. Under Bertrand competition, however, the signaling effect works into the opposite direction. Firms with low costs reduce their bids by more than firms with high costs, so that it becomes unclear which types will submit the highest bids. As Proposition 2 shows, for $n > 2$ the two opposing effects prevent incentive compatible mechanisms that choose firms with the lowest costs as winners from existing. In addition, the following corollary shows that separating equilibria are, in fact, impossible for all auctions where the highest bidders win.

Corollary 2 *Consider auctions where the highest bidders win and all bids are revealed. For Bertrand competition and $n > 2$, there does not exist any auction mechanism that has a separating equilibrium.*

Proof. In a separating equilibrium, firms are able to directly infer types from bids. Hence, (continuous) equilibrium bidding strategies have to be either strictly decreasing or strictly increasing. For strictly decreasing strategies, the corresponding direct mechanism is $\langle m, I^{ab} \rangle$. Under Bertrand competition and $n > 2$, Proposition 2 shows that $\langle m, I^{ab} \rangle$ can never be incentive compatible. For strictly increasing strategies the corresponding direct mechanism is similar to $\langle m, I^{ab} \rangle$ but with the allocation rule choosing the firms with the highest costs as winners. Note that in terms of firm i's objective (3), the difference between $\langle m, I^{ab} \rangle$ and this alternative direct mechanism is just that W and L are interchanged and k is replaced by $n - k$. With those changes, Lemma 1 continues to hold. Accordingly, the direct mechanism choosing the firms with the highest costs as winners is incentive compatible only if

$$\begin{aligned} &-\left(\Pi_1^{L\prime}(c,c,c) - \Pi_1^{W\prime}(c,c,c)\right) g_{n-k}(c) \\ &\qquad + \int_c^{\overline{c}} \Pi_{12}^{L\prime\prime}(c,c,z_{n-k}) g_{n-k}(z_{n-k}) dz_{n-k} + \int_{\underline{c}}^{c} \Pi_{12}^{W\prime\prime}(c,c,z_{n-k}) g_{n-k}(z_{n-k}) dz_{n-k} \geq 0. \end{aligned}$$

Under Bertrand competition, this simplifies to

$$-\left\{\frac{1}{2} - \delta_2 + \delta_1\right\} \varepsilon g_{n-k}(c) - \delta_2 \geq 0$$

which is clearly violated. ■

As under Bertrand competition separating equilibria might only exist as a special case, we will focus on Cournot competition for the rest of this section. For the three auction formats we have introduced in Subsection 3.2 equilibrium bidding strategies are given in Proposition 1. From Proposition 2 we know that the corresponding direct mechanism is incentive compatible under Cournot competition. In order to make sure that such a separating equilibrium actually exists for each auction, we are left to verify that equilibrium strategies β_A, β_D, and β_U are strictly decreasing in c.

As we show below expected payment $m(c)$ is strictly decreasing in c under Cournot competition. With each firm having to pay its bid, the all-pay auction therefore has a separating equilibrium. In a discriminatory or a uniform-price auction, losers are not asked to pay anything. But with all bids being announced, the losers' bids can still work as a signaling device. Especially for firms with very high costs that are almost sure that they will not be amongst the winners, the credibility of signals becomes very questionable. Not surprisingly, additional assumptions are therefore needed to ensure the existence of a separating equilibrium.

Proposition 3 *Suppose all bids are announced and there is Cournot competition in the second stage. Then, the all-pay auction generally has a separating equilibrium where firms bid according to β_A. For the discriminatory and the uniform-price auction a separating equilibrium where firms bid according to β_D and β_U, respectively, exists only if $k = n-1$. If, in addition to $k = n-1$, $F(c)$ is logconcave and $F(c)^{n-1}$ is convex, then both auction formats have such a separating equilibrium.*

Proof. See Appendix A.3. ■

As shown in Proposition 3, there generally is a separating equilibrium for Cournot competitors bidding in an all-pay auction. In the example of lobbying for subsidies, firms hence increase their lobbying expenses in order to signal their strength, given that those expenses are disclosed. The all-pay auction even has a separating equilibrium if $\varepsilon = 0$, i.e., if there is no cost reduction for the winners. In this case, bidding in the all-pay auction corresponds exactly to the truth-telling equilibrium of Ziv (1993). Our result therefore extends Ziv's finding to a Cournot market with more than two firms and heterogeneous goods.

For the discriminatory and the uniform-price auction credible signaling leading to a complete separation of types is only possible if there is only one loser. Yet $k = n-1$ alone does not guarantee the existence of a separating equilibrium. Proposition 3 provides a sufficient condition requiring the distribution function F to be logconcave but not "too concave". Of course, the credibility problem of losing bids is mitigated if the auctioneer refrains from revealing all losing bids. As we find in the next section, this lets separating equilibria become possible also for discriminatory and uniform-price auctions where $k < n-1$.

6 Revealing the Prices Paid

The auctioneer publicly announcing the amount each firm has to pay has different implications for the three auction formats. In the discriminatory auction the winning bids are revealed, whereas in the uniform-price auction the highest losing bid is announced. In an all-pay auction, announcing the prices to be paid is, of course, equivalent to revealing all bids which is the topic of Section 5. Hence, we exclusively focus in this section on the discriminatory and the uniform-price auction, treating each of the two formats separately.

As we have seen in Section 4, a firm's expected profits π^W and π^L do not actually depend on the elements of $\boldsymbol{\zeta}$ but only on their sum. In the following it is therefore useful to define $S(x, \mathbf{z}) := \sum_{j=1}^{n-1} \zeta_j(x, \mathbf{z})$ and to write $\pi^t(c, \xi(x, \mathbf{z}), S(x, \mathbf{z}))$ rather than $\pi^t(c, \xi(x, \mathbf{z}), \boldsymbol{\zeta}(x, \mathbf{z}))$ for $t = W, L$.

6.1 The Discriminatory Auction

For the discriminatory auction, revealing the prices paid is equivalent to announcing all winning bids. The auction corresponds therefore to the direct mechanism $\langle m, I^{wb} \rangle$. Using the slightly changed notation with S instead of $\boldsymbol{\zeta}$, we have, for $t = W, L$,

$$\Pi^t(c, x, z_k) = E\left[\pi^t(c, \xi^{wb}(x, \mathbf{Z}), S^{wb}(x, \mathbf{Z})) \,|\, Z_k = z_k\right]$$

with $\xi^{wb}(x, \mathbf{z})$ given in (11) and

$$S^{wb}(x, \mathbf{z}) = \begin{cases} S^I(\mathbf{z}_{1,\ldots,k-1}) & \text{for} \quad x < z_{k-1} \\ S^{II}(x, \mathbf{z}_{1,\ldots,k-1}) & \text{for} \quad z_{k-1} < x < z_k \\ S^{III}(\mathbf{z}_{1,\ldots,k}) & \text{for} \quad z_k < x \end{cases}$$

where

$$\begin{aligned} S^I(\mathbf{z}_{1,\ldots,k-1}) &:= \sum\nolimits_{j=1}^{k-1} z_j + (n-k)\, E\left[C \,|\, C > z_{k-1}\right] \\ S^{II}(x, \mathbf{z}_{1,\ldots,k-1}) &:= \sum\nolimits_{j=1}^{k-1} z_j + (n-k)\, E\left[C \,|\, C > x\right] \\ S^{III}(\mathbf{z}_{1,\ldots,k}) &:= \sum\nolimits_{j=1}^{k} z_j + (n-k-1)\, E\left[C \,|\, C > z_k\right] \end{aligned}$$

Note that in the case firm i belongs to the winners, i.e., i expects to receive Π^W, we have $x < z_k$. Similarly, for Π^L there is $x > z_k$. Therefore,

$$\Pi^L(c, x, z_k) = E\left[\pi^L(c, E[C \,|\, C > Z_k], S^{III}(\mathbf{Z}_{1,\ldots,k})) \,|\, Z_k = z_k\right]$$

and

$$\begin{aligned}
&\Pi^W(c, x, z_k) = \\
&\quad \int_x^{z_k} \int_{\underline{c}}^{z_{k-1}} \ldots \int_{z_{k-3}}^{z_{k-1}} \pi^W(c, x, S^I(\mathbf{z}_{1,\ldots,k-1})) \frac{g_{1,\ldots,k}(\mathbf{z}_{1,\ldots,k})}{g_k(z_k)} dz_{k-2} \ldots dz_1 dz_{k-1} \\
&\quad + \int_{\underline{c}}^{x} \int_{\underline{c}}^{z_{k-1}} \ldots \int_{z_{k-3}}^{z_{k-1}} \pi^W(c, x, S^{II}(x, \mathbf{z}_{1,\ldots,k-1})) \frac{g_{1,\ldots,k}(\mathbf{z}_{1,\ldots,k})}{g_k(z_k)} dz_{k-2} \ldots dz_1 dz_{k-1}
\end{aligned}$$

or

$$\begin{aligned}
\Pi^W(c, x, z_k) &= \Pr[x < Z_{k-1}, Z_k = z_k] E\left[\pi^W(c, x, S^I(\mathbf{Z}_{1,\ldots,k-1})) \,|\, x < Z_{k-1}, Z_k = z_k\right] \\
&\quad + \Pr[Z_{k-1} < x, Z_k = z_k] E\left[\pi^W(c, x, S^{II}(x, \mathbf{Z}_{1,\ldots,k-1})) \,|\, Z_{k-1} < x, Z_k = z_k\right]. \quad (23)
\end{aligned}$$

Employing the results of Lemmata 3 and 4, Lemma 1 implies the following.

Proposition 4 *Under Cournot competition, the direct mechanism $\langle m, I^{wb} \rangle$ is incentive compatible. Under Bertrand competition, $\langle m, I^{wb} \rangle$ is incentive compatible iff $k = 1$ and*

$$\left\{\frac{1}{2} - \delta_2 + \delta_1\right\} \varepsilon - \delta_2 \left(E\left[C \,|\, C > c\right] - c\right) - \delta_2 \frac{1 - F(c)}{(n-1) f(c)} \geq 0 \quad (24)$$

for all $c \in [\underline{c}, \overline{c}]$.

Proof. See Appendix A.4. ■

As in the case where all bids are revealed, under Cournot competition separating equilibria are generally possible. In addition, revealing less information opens up the possibility of a separating equilibrium in the case where there is a single winner even under Bertrand competition. Indeed, $k = 1$ corresponds to the mechanism with the least information revealed among all direct mechanisms $\langle m, I^{wb} \rangle$.

In addition to the corresponding direct mechanism being incentive compatible, the bidding strategy β_D has to be strictly increasing in order for the discriminatory auction to have a separating equilibrium. Checking this second condition represents our next task. At this point, we impose the additional assumption that the density f is logconcave which enables us to obtain the following result.

Proposition 5 *Suppose f is logconcave. Under Cournot competition, there exists an $a^* \in \mathbb{R}$ such that the discriminatory auction with the winners' bids revealed has a separating equilibrium where firms bid according to β_D if the market size $a \geq a^*$. Under Bertrand competition and with $k = 1$, there is a $d^* \in (0, 1]$ so that given $\varepsilon > 0$ such a separating equilibrium exists for all $d \leq d^*$.*

Proof. See Appendix A.5. ■

Revealing only the winning bids in a discriminatory auction eliminates the problem of noncredible signaling through losing bids that have no costly consequences for the senders. At the same time, however, an other issue arises because of what we identified as the second signaling effect in Subsection 3.4. Consider the event that firm i submits the lowest winning bid while pretending to have costs x. Increasing x leads to an increase in the costs that losing firms are generally believed to have which in turn increases i's expected profits. This effect therefore provides an incentive to reduce bids. In the discriminatory auction the effect is strongest for high cost firms such that it increases the slope of the bidding strategy β_D. Under Cournot competition both the value of winning the auction and the first signaling effect support a decreasing β_D. According to Proposition 5, a sufficiently big market is enough to ensure that the second signaling effect does not dominate the other two effects.[9]

Under Bertrand competition and $k = 1$, there is a separating equilibrium if the goods are sufficiently heterogeneous, i.e. if competition among firms is not too fierce. When the heterogeneity of goods is increased winning the auctions gains in importance relative to the signaling incentives, such that at some point a separating equilibrium exists. Interestingly, as we show in Appendix A.5, $\beta_D' < 0$ implies that condition (24) is fulfilled. Hence, it generally cannot be the case that $\langle m, I^{wb} \rangle$ fails to be incentive compatible although β_D is strictly decreasing.

6.2 The Uniform-price Auction

In the uniform-price auction, announcing the prices winners have to pay means that the highest losing bid is revealed. Accordingly, again using the slightly different notation with S instead of $\boldsymbol{\zeta}$, we have, for $t = W, L$,

$$\Pi^t(c, x, z_k) = E\left[\pi^t(c, \xi^{hlb}(x, \mathbf{Z}), S^{hlb}(x, \mathbf{Z})) \mid Z_k = z_k\right]$$

[9] Note that this result for the case of Cournot competition also holds for $\varepsilon = 0$. Even if winning a discriminatory auction with the winners' bids revealed does not provide a direct advantage, firms still participate, exclusively using their bids for signaling.

where $\xi^{hlb}(x, \mathbf{z})$ is given in (13) and

$$S^{hlb}(x, \mathbf{z}) = \begin{cases} S^{I}(z_k) & \text{for} \quad x < z_k \\ S^{II}(x) & \text{for} \quad z_k < x < z_{k+1} \\ S^{III}(z_{k+1}) & \text{for} \quad z_{k+1} < x. \end{cases}$$

with

$$\begin{aligned} S^{I}(z_k) &:= (k-1)\, E\left[C \,|\, C < z_k\right] + z_k + (n-k-1)\, E\left[C \,|\, C > z_k\right], \\ S^{II}(x) &:= kE\left[C \,|\, C < x\right] + (n-k-1)\, E\left[C \,|\, C > x\right], \\ S^{III}(z_{k+1}) &:= kE\left[C \,|\, C < z_{k+1}\right] + z_{k+1} + (n-k-2)\, E\left[C \,|\, C > z_{k+1}\right]. \end{aligned}$$

Note that for Π^W we have always $x < z_k$ and for Π^L there is $x > z_k$. Therefore,

$$\Pi^W(c, x, z_k) = \pi^W(c, E[C \,|\, C < z_k], S^I(z_k))$$

and

$$\begin{aligned} \Pi^L(c, x, z_k) = &\int_x^{\overline{c}} \pi^L(c, x, S^{II}(x)) \frac{g_{k,k+1}(z_k, z_{k+1})}{g_k(z_k)} dz_{k+1} \\ &+ \int_{z_k}^{x} \pi^L(c, E[C \,|\, C > z_{k+1}], S^{III}(z_{k+1})) \frac{g_{k,k+1}(z_k, z_{k+1})}{g_k(z_k)} dz_{k+1}. \end{aligned}$$

Combining Lemmata 3 and 4 with Lemma 1 one more time, we find the following.

Proposition 6 *Under Cournot competition, the direct mechanism $\langle m, I^{hlb} \rangle$ is incentive compatible. Under Bertrand competition, $\langle m, I^{hlb} \rangle$ is incentive compatible iff*

$$\begin{aligned} &\left\{ \frac{1}{2} - \delta_2 + \delta_1 \right\} \varepsilon - \delta_2 \left(c - E\left[C \,|\, C < c\right]\right) \\ &\qquad - \delta_2 \left(E\left[C \,|\, C > c\right] - c\right) \frac{n-k-1}{k} \frac{F(c)}{1-F(c)} - \delta_2 \frac{F(c)}{kf(c)} \geq 0. \quad (25) \end{aligned}$$

for all $c \in [\underline{c}, \overline{c}]$.

Proof. See Appendix A.6. ■

Like for the other two announcement rules, the corresponding direct mechanism continues to be incentive compatible under Cournot competition when just the highest losing bid of the auction is revealed. Under Bertrand competition, revealing only one cost parameter confines the signaling effect enough for separating equilibria to become possible

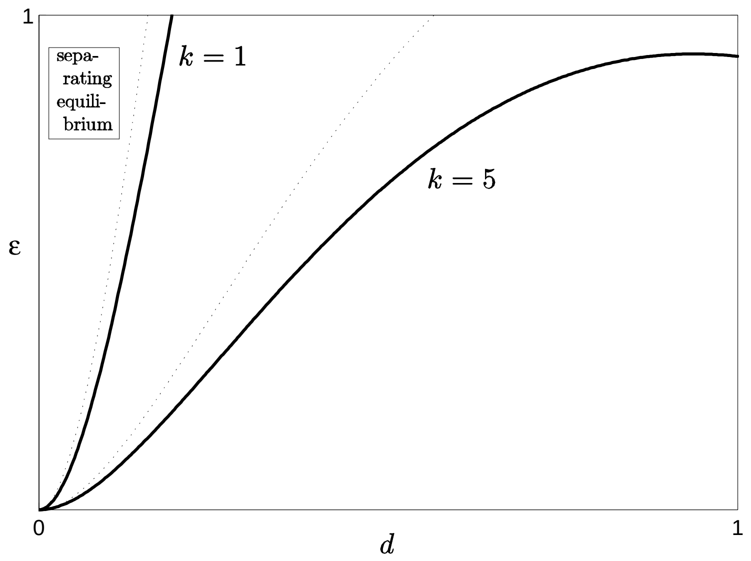

Figure 1: Existence of a separating equilibrium for the uniform-price auction under Cournot competition with uniformly distributed costs.

for all k and n, given condition (25) holds.

In order for a separating equilibrium to exist for the uniform-price auction with the highest losing bid revealed, the corresponding equilibrium bidding strategy must, of course, be a strictly decreasing function. As the one bid that is announced does not directly involve any costs to the submitting bidder, the problem of noncredible signals is also present in this auction, although its impact, as we shall see immediately, is less grave than when all bids are revealed.

Proposition 7 *Suppose $\varepsilon > 0$. Then, under both Cournot and Bertrand competition, there exists a $d^* \in (0, 1]$ such that for all $d \leq d^*$ the uniform-price auction with the highest losing bid revealed has a separating equilibrium where firms bid according to β_U.*

Proof. See Appendix A.7. ■

For both types of competition, incentives stemming from the possibility of using bids as signals might prevent a complete separation of types. For sufficiently heterogenous goods, however, the benefit from winning the cost reduction dominates those counteractive effects, such that there is a separating equilibrium for the uniform-price auction under both Cournot as well as Bertrand competition.

As an example, suppose there are $n = 6$ firms with marginal costs that are uniformly distributed on $[1, 2]$. Moreover, let $a = 12$ and either $k = 1$ or $k = 5$. For the case of

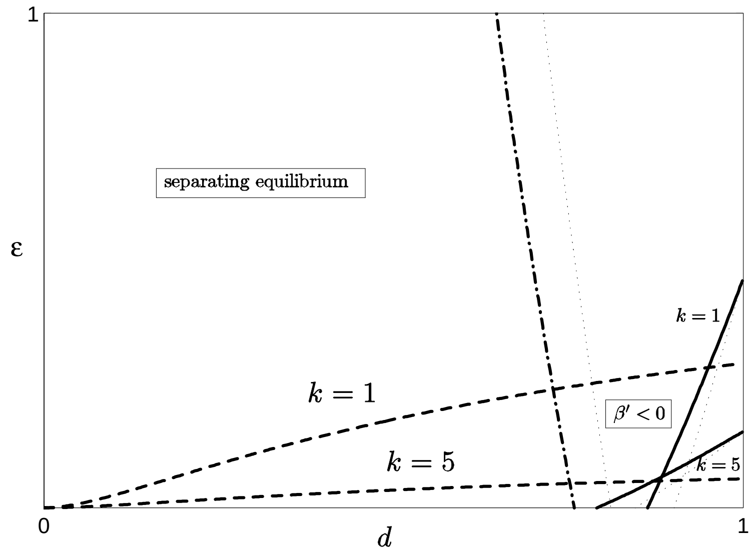

Figure 2: Existence of a separating equilibrium for the uniform-price auction under Bertrand competition with uniformly distributed costs.

Cournot competition Figure 1 displays the combinations of the remaining free parameters d and ε that allow for a separating equilibrium. At all points that lie to the left of the solid line corresponding to $k = 1$ and $k = 5$, respectively, we have $\beta'_U(c) < 0$ implying that the uniform-price auction has a separating equilibrium where firms bid according to β_U. The dotted lines represent an increase of the market size to $a = 16$. Observe that increasing the number of winners enlarges the set of points supporting a separating equilibrium, whereas increasing the market sizes reduces this set.

Figure 2 illustrates the example under Bertrand competition. Here, points where $\beta'_U(c) < 0$ again have to lie to the left of the solid lines. In addition, for a separating equilibrium to exist, condition (25) must be met which is the case for points above (and left of) the dashed lines. Recall that for the case of Bertrand competition we have made the additional assumption (17). Points in the d-ε-plane consistent with this assumption lie to the left of the dash-dotted line. Interestingly, for this uniform example, assumption (17) is sufficient to ensure $\beta'_U(c) < 0$. The dotted lines again represent the situation when $a = 16$. Increasing the market size relaxes the restriction on parameters because of assumption (17) whereas it leaves the requirement for the corresponding direct mechanism to be incentive compatible unchanged.

Under Cournot and under Bertrand competition, existence of the separating equilibrium requires competition among firms to be not too fierce. This can also be formulated as the need for a large enough cost reduction ε. Both, decreasing d and increasing ε, let the advantage for a firm when winning the auction increase compared to the signaling incentives.

7 Conclusion

We study the behavior of bidders in an auction who, after the auction, form an oligopoly and compete to sell their products. Bidders are firms that have private information about their cost structure and take part in the auction in order to win a cost reducing technology. As bids may be observed in the auction process, they can also be used to send signals. We examine three different auction formats. Given the same announcement policy is applied, in a separating equilibrium all three formats are revenue equivalent. However, whether a separating equilibrium actually exists depends amongst other things also on which type of auction is used.

Under Cournot competition, the all-pay auction with all bids revealed and the discriminatory auction with the winning bids revealed both have a fully separating equilibrium, the sole restriction in the latter auction format being that the market size has to be big enough (assuming the probability density is logconcave). The reason for a complete separation of types to arise very generally in those cases is that the firms whose signals are actually observed by their rivals are exactly the firms that have to pay their bid. This way, bids can serve as credible signals. In the uniform-price auction where the price winners have to pay is announced, i.e., where the highest losing bid is revealed, the single firm that actually sends a signal, does not pay anything. In this case, as an additional condition, the benefit from winning the auction has to be sufficiently high (relative to the benefit from signaling) in order for a separating equilibrium to arise. If all bids of a discriminatory or a uniform-price auction are revealed, the problem of noncredible signals becomes more grave, so that separating equilibria only exist in the special case where the auction has only one loser.

Bertrand competition in the second stage constitutes an additional obstacle for the existence of a separating equilibrium. Here, the low-cost firms that profit most from winning the cost reduction are at the same time also the firms with the strongest signaling incentive to reduce their bids in order to understate their costs. Consequently, if all bids are disclosed, separating equilibria are generally impossible when there are more than two firms involved. Revealing less information reduces the weight of the signaling incentive and opens up the possibility of a separating equilibrium under Bertrand competition. If there is only one winner in the discriminatory auction (i.e., if it is a first-price auction) and if only this winning bid is revealed, then a separating equilibrium might exist. The same is true for all uniform-price auctions where only the highest losing bid is revealed. In both cases, it is important that the incentive to win the auction because of its intrinsic value outweighs the counteractive signaling effect. This is generally the case if competition among firms is, thanks to product differentiation, not too fierce, or, alternatively, if the cost reduction is relatively big.

A Appendix

A.1 Proof of Lemma 3

The first order condition to (18) yields

$$q_i(c_i) = \frac{1}{2}\left(a - d\sum_{j\neq i} E[q_j(C_j)\,|\,\mathcal{I}] - c_i\right). \tag{A1}$$

Taking expectations and summing over all $i \neq h$ we obtain

$$\sum_{i\neq h} E[q_i(C_i)\,|\,\mathcal{I}] = \frac{1}{2}\left((n-1)\,a - d\sum_{i\neq h}\sum_{j\neq i} E[q_j(C_j)\,|\,\mathcal{I}] - \sum_{i\neq h} E[C_i\,|\,\mathcal{I}]\right)$$

which is equivalent to

$$\sum_{i\neq h} E[q_i(C_i)\,|\,\mathcal{I}] = \frac{1}{2}\Bigg((n-1)\,a - (n-2)\,d\sum_{j\neq h} E[q_j(C_j)\,|\,\mathcal{I}] \\ - (n-1)\,dE[q_h(C_h)\,|\,\mathcal{I}] - \sum_{i\neq h} E[C_i\,|\,\mathcal{I}]\Bigg).$$

After substituting expectations of (A1) for $E[q_h(C_h)\,|\,\mathcal{I}]$ this can be rearranged to

$$\sum_{j\neq i} E[q_j(C_j)\,|\,\mathcal{I}] = 4\gamma\left(\frac{2-d}{2}(n-1)\,a - \sum_{j\neq i} E[C_j\,|\,\mathcal{I}] + (n-1)\,\frac{d}{2}E[C_i\,|\,\mathcal{I}]\right) \tag{A2}$$

where $\gamma := \frac{1}{2(4-d^2(n-1)+2d(n-2))} > 0$. Substituting (A2) into (A1) and making some further rearrangements we finally obtain the quantity firm i chooses in equilibrium:

$$q_i(c_i) = (4-2d)\,\gamma a + 2d\gamma\sum_{j\neq i} E[C_j\,|\,\mathcal{I}] - d^2\,(n-1)\,\gamma E[C_i\,|\,\mathcal{I}] - \frac{1}{2}c_i.$$

Let $\gamma_0 := (4-2d)\,\gamma a$, $\gamma_1 := 2d\gamma$, and $\gamma_2 := d^2\,(n-1)\,\gamma$. Recall that $E[C_i\,|\,\mathcal{I}] = \xi$ and note $\sum_{j\neq i} E[C_j\,|\,\mathcal{I}] = \sum_{j=1}^{n-1} E[Z_j\,|\,\mathcal{I}] = \sum_{j=1}^{n-1} \zeta_j$. Thus, the expected profit of firm i in the second stage amounts to

$$\pi(c_i, \xi, \boldsymbol{\zeta}) = q(c_i)^2 = \left(\gamma_0 + \gamma_1 \sum_{j=1}^{n-1} \zeta_j - \gamma_2 \xi - \frac{1}{2} c_i\right)^2. \tag{A3}$$

In case firm i belongs to the winners of the auction its marginal costs are reduced by ε, i.e. we have to replace ξ and c_i by $\xi - \varepsilon$ and $c_i - \varepsilon$. In this case also $k-1$ of i's competitors have access to the new technology such that $\sum_{j=1}^{n-1} \zeta_j$ is reduced by $(k-1)\,\varepsilon$. On the other hand, if firm i does not win, k of its competitors use the new technology. Accordingly we simply replace $\sum_{j=1}^{n-1} \zeta_j$ by $\sum_{j=1}^{n-1} \zeta_j - k\varepsilon$.

A.2 Proof of Lemma 4

Summing the inverse demand (15) over all $i \neq h$ and rearranging we obtain

$$\sum_{i\neq h} p_i = (n-1)\,a - d\,(n-1)\,q_h - (d\,(n-2)+1) \sum_{i\neq h} q_i.$$

Substituting this result into (15) and solving for q_i yields the demand for the good produced by firm i:

$$q_i = \tfrac{1-d}{(1-d)(1+d(n-1))} a + \tfrac{d}{(1-d)(1+d(n-1))} \sum_{j\neq i} p_j - \tfrac{1-d+d(n-1)}{(1-d)(1+d(n-1))} p_i.$$

When we substitute the above result for $Q_i\left(\sum p_j\right)$, (19) becomes

$$p_i(c_i) = \arg\max_{p_i} \left(\tfrac{1-d}{1-d+d(n-1)} a + \tfrac{d}{1-d+d(n-1)} \sum_{j\neq i} E[p_j(C_j)\,|\,\mathcal{I}] - p_i\right)(p_i - c_i)\,.$$

The first order condition yields

$$p_i(c_i) = \alpha + \lambda \sum_{j\neq i} E[p_j(C_j)\,|\,\mathcal{I}] + \frac{1}{2} c_i \tag{A4}$$

where $\alpha := \frac{1-d}{2(1-d+d(n-1))}a$ and $\lambda := \frac{d}{2(1-d+d(n-1))}$. Taking expectations and summing over all $i \neq h$ gives

$$\sum_{i\neq h} E[p_i(C_i)\,|\,\mathcal{I}] = (n-1)\,\alpha + \lambda\,(n-1)\,E[p_h(C_h)\,|\,\mathcal{I}] + \lambda\,(n-2)\sum_{i\neq h} E[p_i(C_i)\,|\,\mathcal{I}] + \frac{1}{2}\sum_{i\neq h} E[C_i\,|\,\mathcal{I}].$$

Substituting for $E[p_h(C_h)|\mathcal{I}]$ and then using the result for $\sum_{i\neq h} E[p_i(C_i)|\mathcal{I}]$ together with (A4) we obtain after some rearranging the price firm i sets in equilibrium:

$$p_i(c_i) = \frac{1}{1-\lambda\,(n-1)}\alpha + \frac{\lambda}{2\,(1+\lambda)\,(1-\lambda\,(n-1))}\sum_{j\neq i} E[C_j\,|\,\mathcal{I}] + \frac{\lambda^2\,(n-1)}{2\,(1+\lambda)\,(1-\lambda\,(n-1))}E[C_i\,|\,\mathcal{I}] + \frac{1}{2}c_i.$$

Let $\delta_0 := \frac{1}{1-\lambda(n-1)}\alpha$, $\delta_1 := \frac{\lambda}{2(1+\lambda)(1-\lambda(n-1))}$, and $\delta_2 := \frac{\lambda^2(n-1)}{2(1+\lambda)(1-\lambda(n-1))}$. Again, recall that $E[C_i\,|\,\mathcal{I}] = \xi$ and $\sum_{j\neq i} E[C_j\,|\,\mathcal{I}] = \sum_{j=1}^{n-1}\zeta_j$. For the expected profits of firm i we have

$$\pi(c_i,\xi,\boldsymbol{\zeta}) = (p_i(c_i)-c_i)^2 = \left(\delta_0 + \delta_1\sum_{j=1}^{n-1}\zeta_j + \delta_2\xi - \frac{1}{2}c_i\right)^2. \tag{A5}$$

The distinction between a firm i that has won or lost the auction is identical to that under Cournot competition.

A.3 Proof of Proposition 3

We start with the all-pay auction. The corresponding equilibrium strategy is $\beta_A(c) = m(c)$, as we know from Proposition 1. We have to show that $\beta'_A(c) < 0$ under Cournot competition. From (8) in combination with (20) and Lemma 3 we obtain, after some simplifications and rearranging,

$$m'(c) = -E\left[\pi^W(c,c,\mathbf{Z}) - \pi^L(c,c,\mathbf{Z})\,|\,Z_k = c\right] g_k(c) - 2\gamma_2\left(E\left[\sqrt{\pi^L(c,c,\mathbf{Z})}\right] + \left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon\,(1-G_k(c))\right) \tag{A6}$$

where we have used the facts that

$$\sqrt{\pi^W(c,x,\mathbf{Z})} = \sqrt{\pi^L(c,x,\mathbf{Z})} + \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\}\varepsilon$$

and

$$\int_{\underline{c}}^{\overline{c}} E\left[H(\mathbf{Z}) \mid Z_k = z_k\right] g_k(z_k) dz_k = E\left[H(\mathbf{Z})\right] \quad \text{for any function } H.$$

Hence, we clearly have $\beta_A'(c) = m'(c) < 0$.

Now, consider the equilibrium strategies for the discriminatory and the uniform-price auction given in Proposition 1. Again, those strategies must be strictly decreasing in a separating equilibrium. Recall $m(\overline{c}) = 0$ and note that

$$\beta_D(\overline{c}) = \lim_{c\to\overline{c}} \frac{m(c)}{1 - G_k(c)} = \lim_{c\to\overline{c}} -\frac{m'(c)}{g_k(c)} = \beta_U(\overline{c}).$$

Observe using (A6) that $-\infty < m'(\overline{c}) < 0$. However, $g_k(\overline{c}) = 0$ if $k < n-1$. Hence, in that case, $\beta_D(\overline{c}) = \beta_U(\overline{c}) \to \infty$ and bidding strategies cannot be strictly decreasing everywhere. For a separating equilibrium, $k = n-1$ is therefore necessary.

Let $k = n-1$. We will next derive sufficient conditions for $\beta_U'(c) < 0$. Due to Corollary 1 these conditions also imply $\beta_D'(c) < 0$. From (A6) we obtain

$$\beta_U(c) = -\frac{m'(c)}{g_{n-1}(c)} = \lambda_1(c) + \lambda_2(c) + \lambda_3(c).$$

where

$$\begin{aligned}
\lambda_1(c) &:= E\left[\pi^W(c,c,\mathbf{Z}) - \pi^L(c,c,\mathbf{Z}) \mid Z_{n-1} = c\right] \\
\lambda_2(c) &:= 2\gamma_2 E\left[\sqrt{\pi^L(c,c,\mathbf{Z})}\right] \frac{1}{g_{n-1}(c)} \\
\lambda_3(c) &:= 2\gamma_2 \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\} \varepsilon \frac{1 - G_{n-1}(c)}{g_{n-1}(c)}
\end{aligned}$$

Sufficient for $\beta_U'(c) < 0$ is $\lambda_i'(c) < 0$ for $i = 1,2,3$. Consider first $\lambda_1(c)$ where we have

$$\lambda_1(c) = 2\left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\} \varepsilon E\left[\sqrt{\pi^L(c,c,\mathbf{Z})} \mid Z_{n-1} = c\right] + \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\}^2 \varepsilon^2$$

where

$$E\left[\sqrt{\pi^L(c,c,\mathbf{Z})} \mid Z_{n-1} = c\right] = \gamma_1 (n-2) E\left[C \mid C < c\right] - \left(\frac{1}{2} + \gamma_2 - \gamma_1\right) c + \gamma_0 - k\gamma_1\varepsilon.$$

Hence, $\lambda_1'(c) < 0$ iff

$$\gamma_1 (n-2) \frac{dE[C \mid C < c]}{dc} - \left(\frac{1}{2} + \gamma_2 - \gamma_1\right) < 0.$$

When we assume F do be logconcave, this condition is fulfilled, because of (1) and $\frac{1}{2} + \gamma_2 - \gamma_1 (n-1) > 0$. Consider $\lambda_2(c)$ and note that

$$\frac{\partial}{\partial c} E\left[\sqrt{\pi^L(c, c, \mathbf{Z})}\right] = \delta_2 - \frac{1}{2} < 0.$$

Therefore, $g_{n-1}'(c) \geq 0$ is sufficient for $\lambda_2'(c) < 0$. Moreover, $g_{n-1}'(c) \geq 0$ ensures $\lambda_3'(c) < 0$ as well. Obviously, convexity of $G_{n-1}(c) = F(c)^{n-1}$ is equivalent to $g_{n-1}'(c) \geq 0$.

A.4 Proof of Proposition 4

Consider first the case of Cournot competition. We have

$$\begin{aligned}
\Pi_1^{W\prime}(c, x, x) - \Pi_1^{L\prime}(c, x, x) &= E\left[\pi_1^{W\prime}(c, x, S^{II}(x, \mathbf{Z}_{1,\ldots,k-1})) \mid Z_{k-1} < x, Z_k = x\right] \\
&\quad - E\left[\pi_1^{L\prime}(c, E[C \mid C > Z_k], S^{III}(\mathbf{Z}_{1,\ldots,k})) \mid Z_k = x\right]
\end{aligned}$$

which implies

$$\begin{aligned}
&\Pi_1^{W\prime}(c, x, x) - \Pi_1^{L\prime}(c, x, x) = \\
&\quad - E\left[\sqrt{\pi^W(c, x, S^{II}(x, \mathbf{Z}_{1,\ldots,k-1}))} - \sqrt{\pi^L(c, E[C \mid C > Z_k], S^{III}(\mathbf{Z}_{1,\ldots,k}))} \mid Z_k = x\right]
\end{aligned}$$

such that

$$\Pi_1^{W\prime}(c, x, x) - \Pi_1^{L\prime}(c, x, x) = -(\gamma_1 + \gamma_2)(E[C \mid C > x] - x) - \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\} \varepsilon.$$

Moreover, $\Pi_{12}^{L\prime\prime}(c, x, z_k) = 0$ and

$$\begin{aligned}
\Pi_{12}^{W\prime\prime}(c, x, z_k) &= \Pr[x < Z_{k-1}, Z_k = z_k] E[\pi_{12}^{W\prime\prime}(c, x, S^I(\mathbf{Z}_{1,\ldots,k-1})) \mid x < Z_{k-1}, Z_k = z_k] \\
&\quad + \Pr[Z_{k-1} < x, Z_k = z_k] E[\pi_{12}^{W\prime\prime}(c, x, S^{II}(x, \mathbf{Z}_{1,\ldots,k-1})) \mid Z_{k-1} < x, Z_k = z_k] + \\
&\Pr[Z_{k-1} < x, Z_k = z_k] E[\pi_{13}^{W\prime\prime}(c, x, S^{II}(x, \mathbf{Z}_{1,\ldots,k-1})) S_1^{II\prime}(x, \mathbf{Z}_{1,\ldots,k-1})) | Z_{k-1} < x, Z_k = z_k]
\end{aligned}$$

which implies

$$\Pi_{12}^{W\prime\prime}(c,x,z_k) = \gamma_2 - \Pr[Z_{k-1} < x, Z_k = z_k]\gamma_1 (n-k) \frac{\partial E[C \mid C > x]}{\partial x}.$$

Thus,

$$\begin{aligned} U_{12}''(c,x) = &\left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\} \varepsilon g_k(x) + (\gamma_1 + \gamma_2)(E[C \mid C > x] - x)\, g_k(x) \\ &+ \int_x^{\overline{c}} \gamma_2 g_k(z_k) dz_k - \int_x^{\overline{c}} \Pr[Z_{k-1} < x, Z_k = z_k]\gamma_1 (n-k) \frac{\partial E[C \mid C > x]}{\partial x} g_k(z_k) dz_k. \end{aligned}$$

Carrying out the integrals yields

$$\begin{aligned} U_{12}''(c,x) = &\left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\} \varepsilon g_k(x) + (\gamma_1 + \gamma_2)(E[C \mid C > x] - x)\, g_k(x) \\ &+ \gamma_2 (1 - G_k(x)) - \gamma_1 (G_{k-1}(x) - G_k(x))(n-k) \frac{\partial E[C \mid C > x]}{\partial x} \end{aligned}$$

since

$$\int_x^{\overline{c}} \Pr[Z_{k-1} < x, Z_k = z_k] g_k(z_k) dz_k = \Pr[Z_{k-1} < x < Z_k] = G_{k-1}(x) - G_k(x).$$

Using the fact that

$$(G_{k-1}(c) - G_k(c)) = g_k(c) \frac{1 - F(c)}{(n-k) f(c)} \tag{A7}$$

and

$$\frac{\partial E[C \mid C > c]}{\partial c} = (E[C \mid C > c] - c) \frac{f(c)}{1 - F(c)} \tag{A8}$$

we obtain

$$U_{12}''(c,x) = \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\} \varepsilon g_k(x) + \gamma_2 (E[C \mid C > x] - x)\, g_k(x) + \gamma_2 (1 - G_k(x)). \tag{A9}$$

Clearly, $U_{12}''(c,x) \geq 0$, i.e., (IC2) is fulfilled implying that under Cournot competition the direct mechanism $\langle m, I^{wb} \rangle$ generally is incentive compatible.

Now, consider the case of Bertrand competition. Replace γ_1 by δ_1 and γ_2 by $-\delta_2$ in (A9) in order to obtain

$$U_{12}''(c,x) = \left\{\frac{1}{2} - \delta_2 + \delta_1\right\} \varepsilon g_k(x) - \delta_2 (E[C \mid C > x] - x)\, g_k(x) - \delta_2 (1 - G_k(x)).$$

If $k > 1$, we have $U''_{12}(c, c) < 0$ as $c \to \underline{c}$. Hence, $k = 1$ is necessary for incentive compatibility. With $k = 1$, the necessary condition $U''_{12}(c, c) \geq 0$ from Lemma 1 simplifies to

$$\left\{\frac{1}{2} - \delta_2 + \delta_1\right\} \varepsilon - \delta_2 \left(E\left[C \mid C > c\right] - c\right) - \delta_2 \frac{1 - F(c)}{(n-1)\, f(c)} \geq 0$$

since $\frac{1-G_1(z)}{g_1(z)} = \frac{1-F(z)}{(n-1)f(z)}$. This condition also implies (IC2).

A.5 Proof of Proposition 5

In order for the discriminatory auction with the winning bids revealed to have a separating equilibrium, the corresponding equilibrium bidding strategy has to be strictly decreasing. As it is more convenient analytically, we will in the following work with strategy β_U rather than β_D. Recall from Corollary 1 that $\beta'_U(c) < 0$ always implies $\beta'_D(c) < 0$. As a first step, we now proof the following lemma concerning the equilibrium bidding strategy of the uniform-price auction where the winning bids are revealed which we denote by $\beta_U^{wb}(c)$.

Lemma A1 *Under Cournot competition, equilibrium strategy $\beta_U^{wb}(c)$ can be simplified as follows:*

$$\begin{aligned}\beta_U^{wb}(c) = {} & \frac{1 - G_{k-1}(c)}{g_k(c)} 2\gamma_2 \Omega^I(c) \\ & + 2\left(\gamma_2 \left\{E[C|C > c] - c\right\} + \gamma_2 \frac{1 - F(c)}{(n-k)\, f(c)} + \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\}\varepsilon\right) \Omega^{II}(c) \\ & - \left(\left(\gamma_1 + \gamma_2\right)\left\{E[C|C > c] - c\right\} + \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\}\varepsilon\right)^2 \quad \text{(A10)}\end{aligned}$$

where

$$\Omega^I(c) := \gamma_0 + \gamma_1 E\left[S^I(\mathbf{Z}_{1,\ldots,k-1}) \mid c < Z_{k-1}\right] - \gamma_2 c - \frac{1}{2}c + \left\{\frac{1}{2} + \gamma_2 - (k-1)\,\gamma_1\right\}\varepsilon$$

and

$$\begin{aligned}\Omega^{II}(c) := {} & \gamma_0 + \gamma_1 \left\{(k-1)\, E\left[C \mid C < c\right] + (n-k)\, E\left[C \mid C > c\right]\right\} \\ & - \gamma_2 c - \frac{1}{2}c + \left\{\frac{1}{2} + \gamma_2 - (k-1)\,\gamma_1\right\}\varepsilon.\end{aligned}$$

If $f(c)$ is logconcave, then $\frac{d\Omega^I(c)}{dc} < 0$ as well as $\frac{d\Omega^{II}(c)}{dc} < 0$.

Proof. Since with only the winning bids revealed $\Pi_2^{L\prime}(c,c,z_k)=0$, we have

$$\beta_U^{wb}(c)=\Pi^W(c,c,c)-\Pi^L(c,c,c)-\frac{1}{g_k(c)}\int_c^{\overline{c}}\Pi_2^{W\prime}(c,c,z_k)g_k(z_k)dz_k.$$

Starting with

$$\begin{aligned}\Pi^W(c,c,c)-\Pi^L(c,c,c)=E\big[&\pi^W(c,c,S^{II}(c,\mathbf{Z}_{1,\dots,k-1}))\\&-\pi^L(c,E[C|C>Z_k],S^{III}(\mathbf{Z}_{1,\dots,k}))\quad|\;Z_k=c\big]\end{aligned}$$

we find after plugging in profits under Cournot competition and engaging in some rearranging

$$\begin{aligned}\Pi^W(c,c,c)-\Pi^L(c,c,c)=\,&2\left((\gamma_1+\gamma_2)\left\{E[C|C>c]-c\right\}+\left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon\right)\Omega^{II}(c)\\&-\left((\gamma_1+\gamma_2)\left\{E[C|C>c]-c\right\}+\left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon\right)^2\end{aligned}$$

where $\Omega^{II}(c)$ is defined in Lemma A1. From (23) we find, using the fact that

$$S^I(\mathbf{z}_{1,\dots,k-2},x)=S^{II}(x,\mathbf{z}_{1,\dots,k-2},x),$$

$$\begin{aligned}\Pi_2^{W\prime}(c,x,z_k)=\,&\Pr[x<Z_{k-1},Z_k=z_k]E\left[\pi_2^{W\prime}(c,x,S^I(\mathbf{Z}_{1,\dots,k-1}))\,|\,x<Z_{k-1},Z_k=z_k\right]\\&+\Pr[Z_{k-1}<x,Z_k=z_k]E\big[\pi_2^{W\prime}(c,x,S^{II}(x,\mathbf{Z}_{1,\dots,k-1}))\\&\quad+\pi_3^{W\prime}(c,x,S^{II}(x,\mathbf{Z}_{1,\dots,k-1}))S_1^{II\prime}(x,\mathbf{Z}_{1,\dots,k-1})\,|\,Z_{k-1}<x,Z_k=z_k\big]\end{aligned}$$

such that

$$\begin{aligned}\int_c^{\overline{c}}\Pi_2^{W\prime}(c,c,z_k)g_k(z_k)dz_k=\,&\Pr[c<Z_{k-1}]E\left[\pi_2^{W\prime}(c,c,S^I(\mathbf{Z}_{1,\dots,k-1}))\,|\,c<Z_{k-1}\right]\\&+\Pr[Z_{k-1}<c<Z_k]E\big[\pi_2^{W\prime}(c,c,S^{II}(c,\mathbf{Z}_{1,\dots,k-1}))\\&\quad+\pi_3^{W\prime}(c,c,S^{II}(c,\mathbf{Z}_{1,\dots,k-1}))S_1^{II\prime}(c,\mathbf{Z}_{1,\dots,k-1})\,|\,Z_{k-1}<c<Z_k\big].\end{aligned}$$

Plugging in Cournot profits and rearranging we obtain

$$\int_c^{\overline{c}} \Pi_2^{W\prime}(c,c,z_k) g_k(z_k) dz_k = -\left(1 - G_{k-1}(c)\right) 2\gamma_2 \Omega^I(c)$$
$$- \left(G_{k-1}(c) - G_k(c)\right) 2 \left(\gamma_2 - \gamma_1 (n-k) \frac{dE\left[C \,|\, C > c\right]}{dc}\right) \Omega^{II}(c)$$

where $\Omega^I(c)$ is defined in Lemma A1. Hence, using (A7) and (A8), we finally obtain (A10).

Assuming f to be logconcave, we immediately find

$$\frac{d\Omega^{II}(c)}{dc} \leq \gamma_1 (n-1) - \gamma_2 - \frac{1}{2} < 0.$$

We are left to show that $\frac{d\Omega^I(c)}{dc} < 0$ as well. Due to the law of iterated expectations

$$\begin{aligned} E\left[S^I(\mathbf{Z}_{1,\dots,k-1}) \,|\, c < Z_{k-1}\right] &= E\left[\textstyle\sum_{j=1}^{n-1} Z_j \,|\, c < Z_{k-1}\right] \\ &= E\left[\textstyle\sum_{j=1}^{k-2} Z_j \,|\, c < Z_{k-1}\right] + (n-k+1)\, E\left[C \,|\, C > c\right]. \end{aligned}$$

Note that

$$\begin{aligned} &\left(1 - G_{k-1}(c)\right) E\left[\textstyle\sum_{j=1}^{k-2} Z_j \,|\, c < Z_{k-1}\right] \\ &\quad = \left(1 - G_{k-2}(c)\right) E\left[\textstyle\sum_{j=1}^{k-2} Z_j \,|\, c < Z_{k-2}\right] \\ &\qquad\qquad + \left(G_{k-2}(c) - G_{k-1}(c)\right) E\left[\textstyle\sum_{j=1}^{k-2} Z_j \,|\, Z_{k-2} < c < Z_{k-1}\right] \\ &\quad = \left(1 - G_{k-2}(c)\right) E\left[\textstyle\sum_{j=1}^{k-3} Z_j \,|\, c < Z_{k-2}\right] + \left(1 - G_{k-2}(c)\right) E\left[Z_{k-2} \,|\, c < Z_{k-2}\right] \\ &\qquad\qquad + \left(G_{k-2}(c) - G_{k-1}(c)\right) (k-2)\, E\left[C \,|\, C < c\right] \\ &\quad = \sum_{s=1}^{k-2} \left\{ \left(1 - G_s(c)\right) E\left[Z_s \,|\, c < Z_s\right] + \left(G_s(c) - G_{s+1}(c)\right) s E\left[C \,|\, C < c\right] \right\} \end{aligned}$$

and therefore

$$E\left[\textstyle\sum_{j=1}^{k-2} Z_j \,|\, c < Z_{k-1}\right] = \sum_{s=1}^{k-2} \omega_s(c)$$

where

$$\omega_s(c) := \frac{1 - G_s(c)}{1 - G_{k-1}(c)} E\left[Z_s \,|\, Z_s > c\right] + \frac{G_s(c) - G_{k-1}(c)}{1 - G_{k-1}(c)} E\left[C \,|\, C < c\right].$$

Taking the derivative, we have

$$\omega_s'(c) = \frac{1-G_s(c)}{1-G_{k-1}(c)} \frac{dE\left[Z_s \,|\, Z_s > c\right]}{dc} + \frac{G_s(c)-G_{k-1}(c)}{1-G_{k-1}(c)} \frac{dE\left[C \,|\, C < c\right]}{dc} + \frac{d}{dc}\left(\frac{1-G_s(c)}{1-G_{k-1}(c)}\right)\left(E\left[Z_s \,|\, Z_s > c\right] - E\left[C \,|\, C < c\right]\right).$$

From Theorem 3.3 in Nanda and Shaked (2001) follows $\frac{d}{dc}\left(\frac{1-G_s(c)}{1-G_{k-1}(c)}\right) \leq 0$ for $s < k-1$. Consequently, for logconcave f, $\omega_s'(c) \leq 1$, and therefore

$$\frac{d\Omega^I(c)}{dc} \leq \gamma_1 (n-1) - \gamma_2 - \frac{1}{2} < 0.$$

■

We are now ready to prove the proposition, separately looking at the Cournot and the Bertrand case.

A.5.1 Cournot competition

According to Proposition 4 the corresponding direct mechanism generally is incentive compatible under Cournot competition. The discriminatory auction therefore has a separating equilibrium if $\frac{d}{dc}\beta_U^{wb}(c) < 0$. From (A10) we obtain

$$\frac{d\beta_U^{wb}(c)}{dc} = H_1(c)\frac{d\Omega^I(c)}{dc} + H_2(c)\frac{d\Omega^{II}(c)}{dc} + \frac{dH_1(c)}{dc}\Omega^I(c) + \frac{dH_2(c)}{dc}\Omega^{II}(c) + \frac{dH_3(c)}{dc}$$

where

$$\begin{aligned} H_1(c) &:= 2\gamma_2 \frac{1-G_{k-1}(c)}{g_k(c)}, \\ H_2(c) &:= 2\left(\gamma_2\left\{E[C|C>c]-c\right\} + \gamma_2\frac{1-F(c)}{(n-k)\,f(c)} + \left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon\right), \\ H_3(c) &:= -\left((\gamma_1+\gamma_2)\left\{E[C|C>c]-c\right\} + \left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon\right)^2. \end{aligned}$$

For logconcave f, Lemma A1 implies $H_1(c)\frac{d\Omega^I(c)}{dc} + H_2(c)\frac{d\Omega^{II}(c)}{dc} < 0$ for all $c \in (\underline{c}, \overline{c})$. Moreover, logconcavity of f also implies logconcavity of $1-G_{k-1}(c)$ which, together with the fact that $\frac{g_{k-1}(c)}{g_k(c)} = \frac{k-1}{n-k}\frac{1-F(c)}{F(c)}$ is decreasing in c, implies that $\frac{1-G_{k-1}(c)}{g_k(c)}$ is decreasing in c. Consequently, for logconcave f, $\frac{dH_1(c)}{dc} \leq 0$ and $\frac{dH_2(c)}{dc} \leq 0$ but $\frac{dH_3(c)}{dc} \geq 0$. Now, note that both Ω^I and Ω^{II} are increasing in γ_0 while H_1, H_2, and H_3 are unaffected by a change in γ_0. γ_0 is in turn increasing in a. Logconcavity of f does not rule out

$\frac{dH_1(c)}{dc} = \frac{dH_2(c)}{dc} = 0$. But note that $\frac{dH_2(c)}{dc} = 0$ implies $\frac{dH_3(c)}{dc} = 0$. Hence, there exists an a^* such that for all $a \geq a^*$

$$-\frac{dH_1(c)}{dc}\Omega^I(c) - \frac{dH_2(c)}{dc}\Omega^{II}(c) \geq \frac{dH_3(c)}{dc}.$$

This condition is sufficient for $\frac{d}{dc}\beta_U^{wb}(c) < 0$ and therefore implies the existence of a separating equilibrium.

A.5.2 Bertrand competition

From Proposition 4 we know that under Bertrand competition a separating equilibrium is possible only if $k = 1$. By adapting (A10) to the Bertrand case and setting $k = 1$, we obtain

$$\frac{d\beta_U^{wb}(c)}{dc} = \tilde{H}_2(c)\frac{d\tilde{\Omega}^{II}(c)}{dc} + \frac{d\tilde{H}_2(c)}{dc}\tilde{\Omega}^{II}(c) + \frac{d\tilde{H}_3(c)}{dc}$$

where

$$\begin{aligned}
\tilde{\Omega}^{II}(c) &:= \delta_0 + \delta_1 (n-1) E[C \mid C > c] + \delta_2 c - \frac{1}{2}c + \left\{\frac{1}{2} - \delta_2\right\}\varepsilon, \\
\tilde{H}_2(c) &:= 2\left(-\delta_2 \{E[C|C > c] - c\} - \delta_2 \frac{1 - F(c)}{(n-1) f(c)} + \left\{\frac{1}{2} - \delta_2 + \delta_1\right\}\varepsilon\right), \\
\tilde{H}_3(c) &:= -\left((\delta_1 - \delta_2)\{E[C|C > c] - c\} + \left\{\frac{1}{2} - \delta_2 + \delta_1\right\}\varepsilon\right)^2.
\end{aligned}$$

Assuming f to be logconcave and observing that $\delta_1 - \delta_2 > 0$, we find

$$\begin{aligned}
&\tfrac{d\tilde{\Omega}^{II}(c)}{dc} \leq \delta_1 (n-1) + \delta_2 - \frac{1}{2} < 0, \\
&\tfrac{d\tilde{H}_2(c)}{dc} = 2\delta_2 \left\{1 - \tfrac{dE[C|C>c]}{dc} - \tfrac{d}{dc}\left(\tfrac{1-F(c)}{f(c)}\right)\tfrac{1}{n-1}\right\} > 0, \\
&\tfrac{d\tilde{H}_3(c)}{dc} = 2\left((\delta_1 - \delta_2)\{E[C|C > c] - c\} + \left\{\tfrac{1}{2} - \delta_2 + \delta_1\right\}\varepsilon\right)(\delta_1 - \delta_2)\left\{1 - \tfrac{dE[C|C>c]}{dc}\right\} > 0.
\end{aligned}$$

Now, note that since $\frac{d\tilde{H}_2(c)}{dc}\tilde{\Omega}^{II}(c) + \frac{d\tilde{H}_3(c)}{dc} > 0$ and $\frac{d\tilde{\Omega}^{II}(c)}{dc} < 0$, $\frac{d\beta_U^{wb}(c)}{dc} < 0$ implies $\tilde{H}_2(c) > 0$. In turn, $\tilde{H}_2(c) > 0$ implies (24). Thus, $\frac{d\beta_U^{wb}(c)}{dc} < 0$ also implies that the corresponding direct mechanism is incentive compatible. $\frac{d\beta_U^{wb}(c)}{dc} < 0$ is therefore sufficient for the separating equilibrium to exist.

Suppose $d = 0$ and hence $\delta_1 = \delta_2 = 0$. In this case, we have $\frac{d\tilde{\Omega}^{II}(c)}{dc} < 0$ and $\tilde{H}_2(c) = \varepsilon$ whereas $\frac{d\tilde{H}_2(c)}{dc} = \frac{d\tilde{H}_3(c)}{dc} = 0$. Given $\varepsilon > 0$, this clearly implies $\frac{d\beta_U^{wb}(c)}{dc} < 0$. With $\frac{d\beta_U^{wb}(c)}{dc}$ being continuous in d, we conclude that there exists a $d^* \in (0, 1]$ such that $\frac{d\beta_U^{wb}(c)}{dc} < 0$ for all $c \in [\underline{c}, \overline{c}]$ if $d \leq d^*$.

A.6 Proof of Proposition 6

Consider first the case of Cournot competition. We find

$$\begin{aligned}\Pi_1^{W\prime}(c,x,x) - \Pi_1^{L\prime}(c,x,x) &= \pi_1^{W\prime}(c, E[C\,|\,C<x], S^I(x)) - \pi_1^{L\prime}(c,x,S^{II}(x)) \\ &= -(\gamma_1+\gamma_2)\left(x - E\left[C\,|\,C<x\right]\right) - \left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon\end{aligned}$$

and

$$\Pi_{12}^{W\prime\prime}(c,x,z_k) = 0$$

as well as

$$\begin{aligned}\Pi_{12}^{L\prime\prime}(c,x,z_k) = &-\left\{\pi_1^{L\prime}(c,x,S^{II}(x)) - \pi_1^{L\prime}(c,E[C\,|\,C>x],S^{III}(x))\right\}\frac{g_{k,k+1}(z_k,x)}{g_k(z_k)} \\ &+\left\{\pi_{12}^{L\prime\prime}(c,x,S^{II}(x)) + \pi_{13}^{L\prime\prime}(c,x,S^{II}(x))S^{II\prime}(x)\right\}\int_x^{\bar{c}}\frac{g_{k,k+1}(z_k,z_{k+1})}{g_k(z_k)}dz_{k+1}\end{aligned}$$

which implies

$$\begin{aligned}&\int_{\underline{c}}^{x}\Pi_{12}^{L\prime\prime}(c,x,z_k)g_k(z_k)dz_k = \\ &\quad-\left\{\pi_1^{L\prime}(c,x,S^{II}(x)) - \pi_1^{L\prime}(c,E[C\,|\,C>x],S^{III}(x))\right\}g_{k+1}(x) \\ &\qquad+\left\{\pi_{12}^{L\prime\prime}(c,x,S^{II}(x)) + \pi_{13}^{L\prime\prime}(c,x,S^{II}(x))S^{II\prime}(x)\right\}\left(G_k(x) - G_{k+1}(x)\right).\end{aligned}$$

In turn, this yields

$$\begin{aligned}\int_{\underline{c}}^{x}\Pi_{12}^{L\prime\prime}(c,x,z_k)g_k(z_k)dz_k = &(\gamma_1+\gamma_2)\left(E\left[C\,|\,C>x\right]-x\right)g_{k+1}(x) \\ &+\left\{\gamma_2 - \gamma_1 S^{II\prime}(x)\right\}\left(G_k(x)-G_{k+1}(x)\right).\end{aligned}$$

Thus,

$$\begin{aligned}U_{12}^{\prime\prime}(c,x) = &\left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon g_k(x) + (\gamma_1+\gamma_2)\left(x - E\left[C\,|\,C<x\right]\right)g_k(x) \\ &+(\gamma_1+\gamma_2)\left(E\left[C\,|\,C>x\right]-x\right)g_{k+1}(x) + \left\{\gamma_2-\gamma_1 S^{II\prime}(x)\right\}\left(G_k(x)-G_{k+1}(x)\right). \quad \text{(A11)}\end{aligned}$$

Consider the following useful fact about order statistics:

$$\left(G_k(c) - G_{k+1}(c)\right) = g_k(c)\frac{F(c)}{kf(c)} = g_{k+1}(c)\frac{1-F(c)}{(n-k-1)\,f(c)}. \qquad \text{(A12)}$$

Moreover, since

$$S^{II\prime}(c) = k\frac{\partial E\left[C\,|\,C<c\right]}{\partial c} + (n-k-1)\,\frac{\partial E\left[C\,|\,C>c\right]}{\partial c}$$

and because of

$$\frac{\partial E\left[C\,|\,C<c\right]}{\partial c} = \left(c - E\left[C\,|\,C<c\right]\right)\frac{f(c)}{F(c)}$$

and (A8), (A11) simplifies to

$$\begin{aligned} U_{12}''(c,x) = &\left\{\frac{1}{2}+\gamma_2+\gamma_1\right\}\varepsilon g_k(x) + \gamma_2\left(x - E\left[C\,|\,C<x\right]\right)g_k(x) \\ &+ \gamma_2\left(E\left[C\,|\,C>x\right]-x\right)g_{k+1}(x) + \gamma_2\left(G_k(x)-G_{k+1}(x)\right). \quad \text{(A13)} \end{aligned}$$

Clearly, $U_{12}''(c,x) \geq 0$, i.e. (IC2) holds. Hence, the direct mechanism $\left\langle m, I^{hlb}\right\rangle$ is incentive compatible under Cournot competition.

Now, consider the case of Bertrand competition. Again, we replace γ_1 by δ_1 and γ_2 by $-\delta_2$ in (A13) in order to obtain

$$\begin{aligned} U_{12}''(c,x) = &\left\{\frac{1}{2}-\delta_2+\delta_1\right\}\varepsilon g_k(x) - \delta_2\left(x - E\left[C\,|\,C<x\right]\right)g_k(x) \\ &- \delta_2\left(E\left[C\,|\,C>x\right]-x\right)g_{k+1}(x) - \delta_2\left(G_k(x)-G_{k+1}(x)\right). \end{aligned}$$

Hence, dividing by $g_k(c)$ and making use of (A12), the necessary condition $U_{12}''(c,c) \geq 0$ for incentive compatibility of $\left\langle m, I^{hlb}\right\rangle$ simplifies to

$$\begin{aligned} &\left\{\frac{1}{2}-\delta_2+\delta_1\right\}\varepsilon - \delta_2\left(c - E\left[C\,|\,C<c\right]\right) \\ &- \delta_2\left(E\left[C\,|\,C>c\right]-c\right)\frac{n-k-1}{k}\frac{F(c)}{1-F(c)} - \delta_2\frac{F(c)}{kf(c)} \geq 0. \end{aligned}$$

Again, this condition is also equivalent to (IC2).

A.7 Proof of Proposition 7

We begin this proof with a lemma that shows some implications of the assumptions on F we have made at the beginning of Section 2.

Lemma A2 *The assumptions $f(c) > 0$ and $f'(c) \in \mathbb{R}$ for all $c \in [\underline{c}, \overline{c}]$ imply that, for all $c \in [\underline{c}, \overline{c}]$,*

$$\frac{F(c)}{f(c)} \in \mathbb{R}_+, \quad \frac{1-F(c)}{f(c)} \in \mathbb{R}_+, \quad \frac{d}{dc}\left(\frac{F(c)}{f(c)}\right) \in \mathbb{R}, \quad \frac{d}{dc}\left(\frac{1-F(c)}{f(c)}\right) \in \mathbb{R},$$

$$\frac{dE[C|C<c]}{dc} \in \mathbb{R}_+, \quad \frac{dE[C|C>c]}{dc} \in \mathbb{R}_+ \tag{A14}$$

$$\textit{and} \quad \frac{d^2E[C\,|\,C>c]}{dc^2} \in \mathbb{R} \tag{A15}$$

Proof. Obviously, $f(c) > 0$ implies $\frac{F(c)}{f(c)} \in \mathbb{R}_+$, $\frac{1-F(c)}{f(c)} \in \mathbb{R}_+$. Moreover, from

$$\frac{d}{dc}\left(\frac{F(c)}{f(c)}\right) = 1 - \frac{F(c)}{f(c)}\frac{f'(c)}{f(c)} \quad \text{and} \quad \frac{d}{dc}\left(\frac{1-F(c)}{f(c)}\right) = -1 - \frac{1-F(c)}{f(c)}\frac{f'(c)}{f(c)}$$

we see that $f'(c) \in \mathbb{R}$ implies $\frac{d}{dc}\left(\frac{F(c)}{f(c)}\right) \in \mathbb{R}$ and $\frac{d}{dc}\left(\frac{1-F(c)}{f(c)}\right) \in \mathbb{R}$.

Let us turn to (A14). Observe that, using integration by parts, one can show

$$E[C|C<c] = c - \frac{\int_{\underline{c}}^{c} F(z)dz}{F(c)} \quad \text{and} \quad E[C|C>c] = c + \frac{\int_{c}^{\overline{c}} (1-F(z))\,dz}{1-F(c)}.$$

Hence,

$$\frac{dE[C|C<c]}{dc} = \frac{\int_{\underline{c}}^{c} F(z)dz}{F(c)^2} f(c) \quad \text{and} \quad \frac{dE[C|C>c]}{dc} = \frac{\int_{c}^{\overline{c}} (1-F(z))\,dz}{(1-F(c))^2} f(c).$$

Clearly, $\frac{dE[C|C<c]}{dc} \in \mathbb{R}_+$ for all $c \in (\underline{c}, \overline{c}]$ and $\frac{dE[C|C>c]}{dc} \in \mathbb{R}_+$ for all $c \in [\underline{c}, \overline{c})$. But what about $c \to \underline{c}$ and $c \to \overline{c}$, respectively? Applying l'Hôpital's rule we find

$$\lim_{c \to \underline{c}} \frac{dE[C|C<c]}{dc} = f(\underline{c}) \lim_{c \to \underline{c}} \frac{F(c)}{2F(c)f(c)} = \frac{1}{2}$$

and

$$\lim_{c \to \overline{c}} \frac{dE[C|C>c]}{dc} = f(\overline{c}) \lim_{c \to \overline{c}} \frac{-(1-F(c))}{-2(1-F(c))\,f(c)} = \frac{1}{2} \tag{A16}$$

such that (A14) indeed holds for all $c \in [\underline{c}, \overline{c}]$. Now, consider

$$\frac{d^2E[C|C>c]}{dc^2} = \frac{2f(c)\int_{c}^{\overline{c}} (1-F(z))\,dz - (1-F(c))^2}{(1-F(c))^3} f(c) + \frac{dE[C|C>c]}{dc}\frac{f'(c)}{f(c)}.$$

We immediately see that $f(c) > 0$ and $f'(c) \in \mathbb{R}$ imply $\frac{d^2 E[C|C>c]}{dc^2} \in \mathbb{R}$ for all $c \in [\underline{c}, \overline{c})$. Applying l'Hôpital's rule and making use of (A16), we obtain

$$\lim_{c \to \overline{c}} \frac{d^2 E\left[C|C > c\right]}{dc^2} = f(\overline{c}) \lim_{c \to \overline{c}} \frac{2f'(c) \int_c^{\overline{c}} (1 - F(z))\, dz}{-3\left(1 - F(c)\right)^2 f(c)} + \frac{1}{2}\frac{f'(\overline{c})}{f(\overline{c})} = \frac{1}{6}\frac{f'(\overline{c})}{f(\overline{c})}$$

which completes the proof of (A15). ■

Let us first focus on the case of Cournot competition. In this case it can be shown that, if the highest losing bid is revealed, equilibrium bidding in the uniform-price auction takes the following form:

$$\beta_U(c) = 2\left\{\gamma_2 \rho_1(c) + \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\}\varepsilon\right\}\sqrt{\pi^L\left(c, c, S^{II}\left(c\right)\right)} + \rho_2(c)^2 - \rho_3(c)^2 \frac{n-k-1}{k}\frac{F(c)\left(1 - F\left(c\right)\right)}{f(c)^2}$$

where

$$\begin{aligned}
\rho_1(c) &:= c - E\left[C|C < c\right] + \left((n-k-1)\frac{dE\left[C\,|\,C > c\right]}{dc} + 1\right)\frac{F\left(c\right)}{kf\left(c\right)}, \\
\rho_2(c) &:= (\gamma_1 + \gamma_2)\left(c - E\left[C|C < c\right]\right) + \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\}\varepsilon, \\
\rho_3(c) &:= (\gamma_1 + \gamma_2)\frac{dE\left[C\,|\,C > c\right]}{dc}.
\end{aligned}$$

Hence, we have

$$\begin{aligned}
\beta_U'\left(c\right) = {} & 2\gamma_2 \rho_1'(c)\sqrt{\pi^L\left(c, c, S^{II}\left(c\right)\right)} \\
& + 2\left\{\gamma_2 \rho_1(c) + \left\{\frac{1}{2} + \gamma_2 + \gamma_1\right\}\varepsilon\right\}\frac{\partial}{\partial c}\sqrt{\pi^L\left(c, c, S^{II}\left(c\right)\right)} + 2\rho_2(c)\rho_2'(c) \\
& - 2\rho_3(c)\rho_3'(c)\frac{n-k-1}{k}\frac{F(c)\left(1 - F\left(c\right)\right)}{f(c)^2} - \rho_3(c)^2\frac{n-k-1}{k}\frac{d}{dc}\left(\frac{F(c)}{f(c)}\frac{1 - F\left(c\right)}{f(c)}\right).
\end{aligned}$$

Noting that

$$\rho_1'(c) = 1 - \tfrac{dE[C|C<c]}{dc} + \left((n-k-1)\tfrac{d^2E[C\,|\,C>c]}{dc^2}\right)\tfrac{F(c)}{kf(c)} + \left((n-k-1)\tfrac{dE[C\,|\,C>c]}{dc} + 1\right)\tfrac{d}{dc}\left(\tfrac{F(c)}{kf(c)}\right),$$

$$\rho_2'(c) = (\gamma_1 + \gamma_2)\left(1 - \tfrac{dE[C|C<c]}{dc}\right), \qquad \rho_3'(c) = (\gamma_1 + \gamma_2)\tfrac{d^2E[C\,|\,C>c]}{dc^2},$$

$$\text{and} \quad \frac{\partial}{\partial c}\sqrt{\pi^L\left(c,c,S^{II}\left(c\right)\right)} = -\left(\tfrac{1}{2}+\gamma_2-\gamma_1\left(k\tfrac{dE[C|C<c]}{dc}+\left(n-k-1\right)\tfrac{dE[C|C>c]}{dc}\right)\right),$$

we find that Lemma A2 implies $\beta_U'(c) \in \mathbb{R}$ for all $c \in [\underline{c},\overline{c}]$. Furthermore, observe that $\beta_U'(c)$ is continuous in γ_0, γ_1, and γ_2. As γ_0, γ_1, and γ_2 are continuous in d, $\beta_U'(c)$ is also continuous in d.

Now consider the case of Bertrand competition. We can simply reuse the results for the Cournot case by replacing γ_0, γ_1, and γ_2 with δ_0, δ_1, and $-\delta_2$, respectively. Similarly, we obtain that $\beta_U'(c) \in \mathbb{R}$ and that $\beta_U'(c)$ is continuous in d.

Setting $d = 0$ implies $\gamma_1 = \gamma_2 = \delta_1 = \delta_2 = 0$. Consequently, under both Cournot and Bertrand competition we have $\beta_U'(c) = -\frac{1}{2}\varepsilon$ if $d = 0$. Because $\beta_U'(c)$ is finite and continuous in d, we conclude that, given $\varepsilon > 0$, there exists a $d^* \in (0,1]$ such that for all $d \leq d^*$, $\beta_U'(c) < 0$.

According to Proposition 6, $\beta_U'(c) < 0$ is enough to guarantee the existence of a separating equilibrium for the case of Cournot competition. Yet under Bertrand competition, in addition to $\beta_U'(c) < 0$, (25) must also hold. Using (A8), (25) is equivalent to

$$\left\{\tfrac{1}{2}-\delta_2+\delta_1\right\}\varepsilon-\delta_2\left(c-E\left[C\,|\,C<c\right]\right)-\delta_2\left(\left(n-k-1\right)\tfrac{dE[C\,|\,C>c]}{dc}+1\right)\tfrac{F(c)}{kf(c)} \geq 0.$$

From Lemma A2, the LHS of this inequality is bigger than $-\infty$ for all $c \in [\underline{c},\overline{c}]$. Moreover, it is continuous in d. For $d = 0$, (25) is equivalent to requiring $\varepsilon \geq 0$. Consequently, there is a $d^* \in (0,1]$ such that (25) is fulfilled for all $d \leq d^*$.

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